Department of Physics, IIT Patna

EP302 - Quantum Mechanics II
(Instructor: Sandeep Aashish)

Spring 2024 February 5, 2024

Assignment 1

**Due on Mon 12/2/2024.*

1. Show that for the function $W(E) = \int_{x'}^{x} dx'' \sqrt{2m(E - V(x''))}$, dW/dE is the time t taken to go from start to finish at a constant energy E.

2. Read the Tunneling amplitudes topic from R. Shankar Sec. 16.2. Based on this, complete the exercise asking to estimate the mean lifetime of Alpha particles of kinetic energy $4.2MeV$ tunneling out of a nucleus of charge $Z = 90$ (after emission). [Exercise 16.2.4].

3. Using the quantization condition of the WKB method, find out the allowed energy levels of the harmonic oscillator.

4. Show that if $H^1(t) = -e\mathcal{E}X/[1 + (t/\tau)^2]$, then to first order in perturbation,

$$P_{0\to 1} = \frac{e^2\mathcal{E}^2\pi^2\tau^2}{2m\omega\hbar} e^{-2\omega\tau}$$

5. A hydrogen atom is in the ground state at $t = -\infty$. An electric field $\vec{E}(t) = \vec{k}\mathcal{E}e^{-t^2/\tau^2}$ is applied until $t = \infty$. Show that the probability that the atom ends up in any of the $n = 2$ states is, to first order,

$$P(n = 2) = \left(\frac{e]}{\hbar}\right)^2 \left(\frac{2^{15}a_0^2}{3^{10}}\right) \pi\tau^2 e^{-\omega^2\tau^2/2}$$

   where $\omega = (E_{2lm} - E_{100})/\hbar$.

6. Consider a system subject to a perturbation $H^1(t) = H^1\delta(t)$. Show that if at $t = 0^-$ the system is in the state $|\, i^0 \,\rangle$, the amplitude to be in a state $|\, f^0 \,\rangle \neq |\, i^0 \,\rangle$ at $t = 0^+$ is, to first order,

$$d_f = \frac{-if}{\hbar} \langle\, f^0 \,|H^1|\, i^0 \,\rangle$$

   What does this result tell you about the nature of change of state of this system? Is is consistent with the idea of perturbation theory to begin with?

7. Solve the integral:

$$\int_{-\infty}^{\infty} e^{-\alpha x^2} e^{i\omega x} dx$$