Department of Physics, IIT Patna

EP302 - Quantum Mechanics II
(Instructor: Sandeep Aashish)

Spring 2024 March 26, 2024

Assignment 1

**Due on Fri 5/4/2024.*

1. Consider a position vector $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$. Re-write this vector in a new "spherical basis" given by the complex unit vectors:

$$\hat{e}_1 = -\frac{\hat{x} + i\hat{y}}{\sqrt{2}}; \quad \hat{e}_0 = \hat{z}; \quad \hat{e}_{-1} = \frac{\hat{x} - i\hat{y}}{\sqrt{2}}$$

2. (a) Derive the generators of infinitesimal rotations about x- and y- axis, similar to that about z-axis.
   (b) From the above, obtain the matrix representation in 3D for both operators and generators.
   (c) Verify explicitly the group algebra of the generators of rotation about (i) y- and z-axes; (ii) z- and x-axes.
   (d) Using the matrix representations, show that $[L_z, L^2] = 0$.

3. Derive the transition amplitude for the stimulated emission, following the derivation of the transition rate for absorption using the semi-classical theory of radiation-matter interaction.

4. Assume that in the Einstein coefficient derivation, the number density (N_b/N_0) follows Bose-Einstein distribution instead of the Maxwell distribution. How will the results change?