The theory of creation of particles.

Recall:

→ Einstein's coefficients: For a system of photon gas (where each photon is characterized by its energy, many such photons — N_i # with energy $\hbar\omega_i$) in equilibrium

→ Coeff. B_{12} & B_{21} (for $2 \to 1$) can be calculated using TDPT.

→ Coeff. A can be obtained from B_{21}.

→ However, these results are not derived from first principles!

* How is a photon created?

The idea of second Quantization.

→ Consider a single-freq. mode of photon, with energy Δ.

→ Excitation ⇒ $\Delta \to \underbrace{2\Delta, 3\Delta, \ldots}$

+1 photon / +Δ energy to the sys. after each excitation.

⟷

~ Q.H.O !!

∴ Treating a system of photons with the same energy Δ, as a QHO with op. $a, a^\dagger$:

$$[a, a^\dagger] = 1$$

Review of algebra of $a, a^\dagger$:

→ Number op., $\hat{N}|n\rangle = n\underbrace{|n\rangle}$ → state with $n\Delta$ energy
⇕
n photons.

$$\Rightarrow \hat{N} a^\dagger|n\rangle = a^\dagger a a^\dagger|n\rangle$$
$$= a^\dagger a^\dagger a|n\rangle + a^\dagger|n\rangle$$
$$= a^\dagger \hat{N}|n\rangle + a^\dagger|n\rangle$$
$$= (n+1) a^\dagger|n\rangle$$

$$\therefore \quad a^\dagger|n\rangle = c|n+1\rangle \qquad (\text{assuming } c \in \mathbb{R})$$

$$\Rightarrow c^2 = \langle n+1|c^2|n+1\rangle = \langle n|a a^\dagger|n\rangle$$
$$= \langle n|(1+a^\dagger a)|n\rangle$$
$$= (n+1)\langle n|n\rangle$$

$$\Rightarrow \quad \boxed{c = \sqrt{n+1}}$$

IIIly $\quad a|n\rangle = c'|n-1\rangle$

$$\Rightarrow c'^2 = \langle n|a^\dagger a|n\rangle = n \Rightarrow \boxed{c' = \sqrt{n}}$$

→ Recall from TDPT: The transition amplitude for absorption / stimulated emission:

$$\Gamma \sim |M|^2 \delta(E_f - E_i)$$

where $M = \langle f | H_{int} | i \rangle$

$\because$ A photon is either created or annihilated,

$$H_{int} \approx H_I^{\dagger} a^{\dagger} + H_I a$$

$a^{\dagger}$, a: cr./ann. op. for photons.

Then the initial state: $|i\rangle \equiv |\underbrace{atom_2}_{\text{atom in excited state}}; \underbrace{n_\omega}_{\text{\# photons with freq. } \omega}\rangle$

Illy final state: $|f\rangle = |\underbrace{atom_1}_{\text{atom in ground st.}}; \underbrace{n_\omega + 1}_{\text{1 photon emitted}}\rangle$.

$$\Rightarrow M_{2\to 1} = \langle atom_1; n_{\omega+1} | (H_I^{\dagger} a^{\dagger} + H_I a) | atom_2; n_\omega \rangle$$

$$= \underbrace{\langle atom_1 | H_I^{\dagger} | atom_2 \rangle}_{\equiv M_{fi}^{\dagger}} \langle n_{\omega+1} | a^{\dagger} | n_\omega \rangle$$

$$+ \langle atom_1 | H_I | atom_2 \rangle \langle n_{\omega+1} | a | n_\omega \rangle$$

$$\Rightarrow M_{2\to1} = M_0^{\dagger}\langle n_\omega+1|n_\omega+1\rangle\sqrt{n_\omega+1}$$

$$\Rightarrow |M_{2\to1}|^2 = |M_0|^2(n_\omega+1)$$

Illy, for absorption:

$$|M_{1\to2}|^2 = |M_0|^2 n_\omega$$ ← H/W

∴ The number density at $|atom_2\rangle$:

$$dn_2 = -\underbrace{|M_{2\to1}|^2 n_2}_{\text{energy/photons lost}} + \underbrace{|M_{1\to2}|^2 n_1}_{\text{energy/photons gained}}.$$

$$\Rightarrow \boxed{dn_2 = -|M_0|^2(n_\omega+1)\,n_2 + |M_0|^2 n_\omega n_1} = -dn_1$$

→ <u>NOTE:</u> This result is for only one freq. ω!

⇓

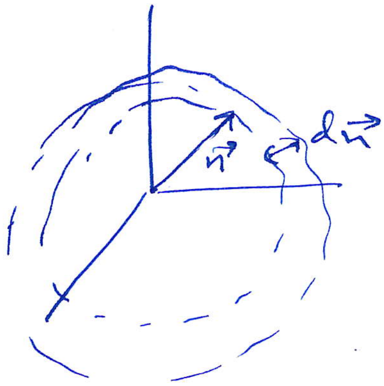
We need to relate to the total intensity for the std. form.

→ Counting of photon states in the range ω & $\omega+d\omega$:

$$\because \Delta = \hbar\omega = \hbar\frac{2\pi}{L}|\vec{n}|$$

$$\Rightarrow \boxed{|\vec{n}| = \cancel{2\pi}\ \frac{L}{2\pi}\,\omega} \Rightarrow \boxed{dn = \frac{L}{2\pi}\,d\omega}$$

In the space of $\vec{n}$:



$$dE(\omega) = \underbrace{d^3\vec{n}}_{\substack{\#\text{ of modes} \\ \text{(volume)}}} \cdot \underbrace{n_\omega}_{\substack{\#\text{density} \\ \text{at }\omega}} \cdot \underbrace{(\hbar\omega)}_{\substack{\text{energy for} \\ 1\text{ photon at }\omega.}}$$

$$\Rightarrow E(\omega) = \int^{\omega} d^3\vec{n}\; n_\omega(\hbar\omega)$$

$$= 4\pi \int dn\, n^2\, n_\omega (\hbar\omega)$$

$$= 4\pi\hbar L^3 \int \frac{d\omega}{(2\pi)^3}\, \omega^3 n_\omega \quad \leftarrow \text{for 1 polarization}$$

$$\Rightarrow \boxed{E(\omega) = \cancel{2}\ 8\pi\hbar L^3 \int \frac{d\omega}{(2\pi)^3}\, \omega^3 n_\omega}$$

$\therefore$ Intensity, $\boxed{I(\omega) = \frac{1}{L^3}\frac{dE}{d\omega} = \frac{\hbar\omega^3}{\pi^2}\, n_\omega}$

$$\Rightarrow \underbrace{dn_2 = -|M_0|^2\left[1 + \frac{\pi^2}{\hbar\omega^3} I(\omega)\right] n_2 + |M_0|^2 \left[\frac{\pi^2}{\hbar\omega^3} I(\omega)\right] n_1}$$

$$\Downarrow$$

$\cancel{B=B}$ $\boxed{B_{12} = B_{21}}$ $\quad \boxed{\frac{A_{21}}{B_{21}} = \frac{\hbar\omega^3}{\pi^2}}$

→ This derivation of Einstein coefficients is one of the first results of QFT!!

→ PS: No assumption of thermal equilibrium!!

→ This method of treating creation & annihilation operators for creation/destruction of photon # is known as the second quantization!

→ Now, the obvious question:

(Q) What is the Schroedinger eq. of such operators?

(Q) What is the formal theory describing creation & annihilation?

⋮

Ans: Quantum Field Theory.

* In general, photons are not the only particles to be created/destroyed..
* Fermions & other bosons are also created/annihilated.
* However, such processes happen at very high energies!

⇒ Any fundamental theory would explain such high energy processes.

~~Naturally~~, Example: LHC

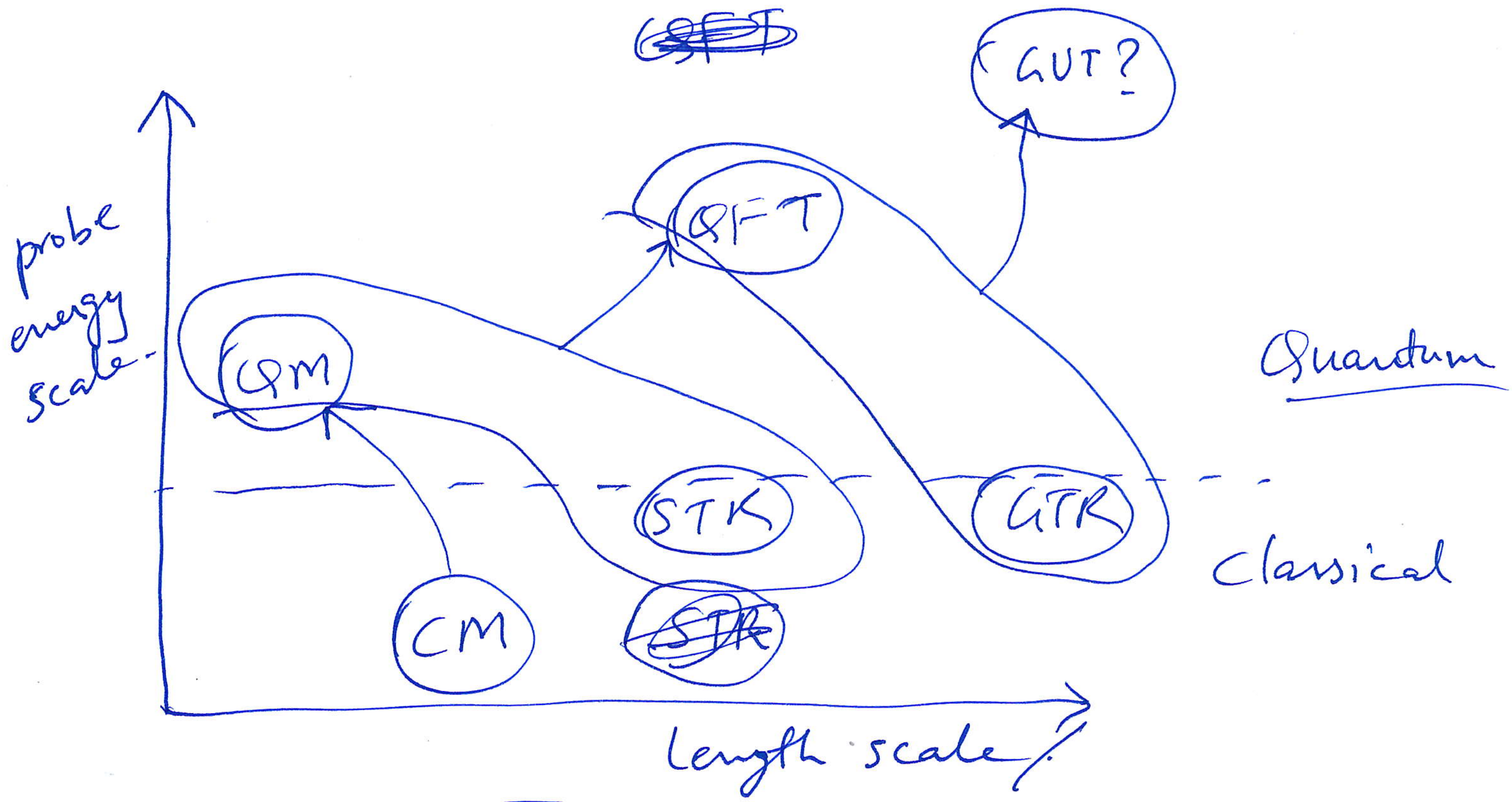
Scattering at high energy.

ples. moving with $v \lesssim c$!

→ Naturally, such a theory must be consistent with STR. — Lorentz invariance.

Q What about TDSE? Is it Lorentz invariant?

$\boxed{\text{H/W}}$



$\boxed{\text{QFT} \rightarrow \text{STR} + \text{QM}}$