EP401/PH521, I-Semester 2024-25, Assignment 1

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1. Consider the coulumb potential (all symbols are as usual in electromagnetic theory):

$$V(r) = -\frac{e^2}{4\pi\epsilon_0 r}$$

   Draw a plot of $V(r)$ using a Mathematica package on a computer.

2. A central potential is defined as $V = V(r) \neq V(\theta, \phi)$. Prove that for a central potential, the wave function is separable in spherical polar coordinates. (*Hint:* Refer to Sec. 2.6 from Bransden)

3. Consider the coordinate space representation of the momentum operator in spherical polar coordinates:

$$\hat{p} = -i\hbar\vec{\nabla} = -i\hbar\left(\hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial\theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\right)$$

   Using the above expression, **derive** the Laplacian operator ∇^2.

4. Consider the time-independent Schroedinger equation:

$$\left(-\frac{\hbar^2}{2m}\hat{p}^2 - V(r)\right)|\,\psi\,\rangle = E|\,\psi\,\rangle$$

   where $V(r)$ is a central potential. What is the Fourier transform of this equation?

5. Recall that during the lectures we came across an effective potential given by:

$$V_{eff}(r) = -\frac{Ze^2}{4\pi\epsilon_0 r} + \frac{l(l+1)\hbar^2}{2\mu r^2}$$

   For $Z = 1$, plot the graph for $V_{eff}(r)$ taking typical values for parameters, using Mathematica.

6. Consider the radial part of the Schroedinger equation for the $u(r)$. How does the equation behave in the limit $r \to 0$? Suppose you introduce a new variable $\tilde{u}(r)$ where $u(r) \equiv r^s\tilde{u}(r)$, such that $\lim_{r\to 0} r^2 V_{eff}(r) = 0$. What is the condition that s has to satisfy in terms of l in the limit $r \to 0$?