Department of Physics, IIT Patna

EP401/PH521 - Atomic and Molecular Physics
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Assignment 2

1. Using the following set of transformations:

$$\vec{A}(\vec{r},t) = \vec{A'}(\vec{r},t) + \vec{\nabla}\chi(\vec{r},t) \tag{1}$$

$$\phi(\vec{r},t) = \phi'(\vec{r},t) - \frac{\partial}{\partial t}\chi(\vec{r},t) \tag{2}$$

$$\psi(\vec{r},t) = \psi'(\vec{r},t)\exp[iq\chi(\vec{r},t)/\hbar] \tag{3}$$

prove that the time dependent Schroedinger equation for a charged particle interacting with a weak classical field given below is covariant:

$$i\hbar\frac{\partial}{\partial t}\psi(\vec{r},t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + i\hbar\frac{q}{2m}(\vec{A}\cdot\vec{\nabla} + \vec{\nabla}\cdot\vec{A}) + \frac{q^2}{2m}\vec{A}^2 + q\phi\right]\psi(\vec{r},t)$$

2. Under what condition the Dipole approximation might break down?

3. Using the result $H_0|\psi_i\rangle = E_i|\psi_i\rangle$ where $H_0 = -(\hbar^2/2m)\nabla^2 - V(r)$, we can write

$$\vec{r}_{ba} = \frac{1}{E_b - E_a}\langle\psi_b|H_0\vec{r} - \vec{r}H_0|\psi_a\rangle$$

Obtain the relation between $\vec{r}_{ba}$ and $\vec{p}_{ba}$ starting from the above equation, where $\vec{p}_{ba} = -i\hbar\vec{\nabla}$.

4. Assuming that the energy level E_a is g_a times degenerate and level E_b is g_b times degenerate, show that the Einstein coefficients satisfy the following relation:

$$g_a B_{ba} = g_b B_{ab}; \quad A_{ab} = \frac{\hbar\omega_{ba}^3}{\pi^2 c^3}B_{ab}.$$

5. Show that the average of $\cos^2\theta$ over all solid angles is $1/3$.

6. What are the allowed transitions from the 2p level in a three level (i.e. $n = 1, 2, 3$) Hydrogenic atom? Compute the transition rate of stimulated emission for an electron in the 2p orbital of Hydrogen atom.