Department of Physics, IIT Patna

EP401/PH521 - Atomic and Molecular Physics
(Instructor: Sandeep Aashish)

Autumn 2024 September 14, 2024

Assignment 3

1. Evaluate the following commutators:
   (a) $[\vec{S}\cdot\vec{L},\vec{L}]$
   (b) $[\vec{S}\cdot\vec{L},\vec{S}]$
   (c) $[\vec{S}\cdot\vec{L},\vec{J}]$
   (d) $[\vec{S}\cdot\vec{L},\vec{L}^2]$
   (e) $[\vec{S}\cdot\vec{L},\vec{S}^2]$
   (f) $[\vec{S}\cdot\vec{L},\vec{J}^2]$

2. Starting from the expression for correction to energy level due to spin-orbit coupling, ΔE_2, obtained in the lectures and taking the value of spin $s = 1/2$, show that ΔE_2 can be rewritten in the following form:

$$-E_n \frac{(Z\alpha)^2}{2nl(l+1/2)(l+1)} \times \begin{cases} l & \text{for } j = l + 1/2 \\ -l-1 & \text{for } j = l - 1/2 \end{cases}$$

3. Show the total energy shift $\Delta E_{nj} = \Delta E_1 + \Delta E_2 + \Delta E_3$ due to relativistic correction, spin-orbit coupling and Darwin term respectively, is given by:

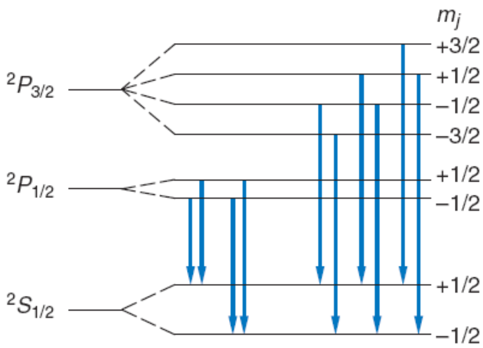
$$\Delta E_{nj} = E_n \frac{(Z\alpha)^2}{n^2}\left(\frac{n}{j+1/2} - \frac{3}{4}\right)$$

4. Using the result for the energy shift due to relativistic correction to the kinetic term, show that the lowest order relativistic correction to the energy levels of the one-dimensional harmonic oscillator is given by,

$$\Delta E = -\frac{(\hbar\omega)^2}{mc^2}\frac{6n^2+6n+3}{32}.$$

5. Compute $\langle 200|r\cos\theta|210\rangle$

6. The magnetic field of the Sun and stars can be determined by measuring the Zeeman-effect splitting of spectral lines. Suppose that the sodium D_1 line emitted in a particular region of the solar disk is observed to be split into the four-component Zeeman effect (see Figure below). What is the strength of the solar magnetic field B in that region if the wavelength difference $\Delta\lambda$ between the shortest and the longest wavelengths is 0.022 nm? (The wavelength of the D_1 line is 589.8 nm.)


m_j
$^2P_{3/2}$
+3/2
+1/2
−1/2
−3/2
$^2P_{1/2}$
+1/2
−1/2
$^2S_{1/2}$
+1/2
−1/2