Department of Physics, IIT Patna

PH623 - Introduction to general relativity and cosmology
(Instructor: Sandeep Aashish)

Spring 2024 January 19, 2024

Assignment 1

**Due on Mon 29/1/2024.*

1. What would qualify as an evidence to support the assertion that the universe is electrically neutral on large scales?

2. Suppose you are a two-dimensional being, living on a sphere of radius R. Show that if you draw a circle of radius r, the circle's circumference will be

$$C = 2\pi R \sin(r/R).$$

   Idealize the Earth as a perfect sphere of radius R = 6371km. If you could measure distances with an error of ± 1 meter, how large a circle would you have to draw on the Earth's surface to convince yourself that the Earth is spherical rather than flat?

3. Write down the invariant length element in Minkowski spacetime, in the spherical polar coordinates for spatial dimensions. What is the metric?

4. Prove that the gradient ($\vec{\nabla}$) is a $\binom{1}{1}$ tensor.

5. From the definition $f_{\alpha\beta} = f(\vec{e}_\alpha, \vec{e}_\beta)$ for the components of a $\binom{0}{2}$ tensor, prove that the transformation law is

$$f'_{\alpha\beta} = \Lambda^{\mu}{}_{\alpha} \Lambda^{\nu}{}_{\beta} f_{\mu\nu}$$

   and that the matrix version of this is

$$(f') = (\Lambda)^T (f)(\Lambda),$$

   where (Λ) is the matrix with components $\Lambda^{\mu}{}_{\alpha}$.

6. Since our definition of a Lorentz frame led us to deduce that the metric tensor has components $\eta_{\alpha\beta}$, this must be true in all Lorentz frames. We are thus led to a more general definition of a Lorentz transformation as one whose matrix $\Lambda^{\mu}{}_{\alpha}$ satisfies:

$$\eta'_{\alpha\beta} = \Lambda^{\mu}{}_{\alpha} \Lambda^{\nu}{}_{\beta} \eta_{\mu\nu}.$$

   Prove that the matrix for a (Lorentz) boost of velocity $v\hat{x}$ satisfies this, so that this new definition includes our older one.