Inflationary Cosmology.

→ A brief history of the universe:
"era" in cosmology

[Big bang — "$t=0$" ← We know nothing!! (almost)

[Inflation (Quant. fluctuation) — $\sim 10^{-35}$ s

[Nucleosynthesis
Baryogenesis } — ~ 1 s → 100 years.
~~"Dark ages"~~

[Recombination (Origin of CMB) — $\sim 400{,}000$ y.

[LSS formation — $\gtrsim 100$ million y.

↓

Today — ~ 14 billion y.

[Dark energy dominated universe] ~~un~~

→ Inflationary cosmology — evolution of Einstein eq. before creation of matter/radiation!

Physics of the Very Early Universe — Inflation.

→ To answer why there's a need for inflation, we revisit the "predictions of past" acc. to the std. Hot Big bang model:

(I) The Flatness problem:

Recall: The Friedmann eq. in terms of the density parameter Ω

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{R_0^2 a^2}$$

$$\Rightarrow 1 - \underbrace{\left(\frac{8\pi G}{3H^2}\right)}_{1/\rho_c \equiv 1/\varepsilon_c}\rho = -\frac{k}{R_0^2 a^2 H^2}$$

$$\Rightarrow 1 - \frac{\rho}{\varepsilon_c} = -\frac{k}{R_0^2 a^2 H^2}$$

$$\Rightarrow \boxed{1 - \Omega = -\frac{k}{R_0^2 a^2 H^2}}$$

At current time:

$$\boxed{1 - \Omega_0 = -\frac{k}{R_0^2 H_0^2}} \qquad (\because a(t_0) = 1)$$

From observations:

$$|1 - \Omega_0| \leq 0.005$$

$$\Rightarrow \left|1 - k\frac{1}{R_0^2 H_0^2}\right| \lesssim 0.005$$

$$\Rightarrow \quad R_0^2 H_0^2 \gtrsim \frac{1}{0.005}$$

$$\Rightarrow \quad \boxed{R_0 \gtrsim \frac{14}{H_0}}$$

Using the value of H_0 in SI unit $(c \neq 1)$:

$$R_0 \gtrsim \frac{14c}{H_0} \approx \frac{14 \times 3 \times 10^8 \, m \, s^{-1}}{10^{-18} \, s^{-1}}$$

$$\boxed{\approx 10^{27} \, m \, !!}$$

Huge!

$\Rightarrow \quad R_0 \gtrsim$ size of the current universe. !!

The puzzle: Why does R_0 have to be so large? OR Why is Ω_0 so close to 1 ??

If $\Omega_0 = 10^5$ or 10^{-5}, doesn't make a difference to physics! ~~So why does nature have such a particular value of Ω_0?~~

~~"Flatness problem"~~

(13) What about $\Omega(t)$ in the past?

$$1-\Omega(t) = -K\left(\frac{1}{R_0 a H}\right)^2 = \frac{H_0^2(1-\Omega_0)}{H^2 a^2}$$

Example, ~~Suppose~~, at $t = 10^9 y \rightarrow$ rad. + matter dominated univ.
$\quad\quad\quad\quad\quad\quad\quad \shortparallel$ t_{rm}

$$\Rightarrow \frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3}$$

$$\Rightarrow 1-\Omega(t) = \frac{(1-\Omega_0)a^2}{\Omega_{r,0}+\Omega_{m,0}}$$

$$\Rightarrow |1-\Omega|_{rad} \propto a^2 \propto t$$

$$|1-\Omega|_{mat} \propto a \propto t^{2/3}$$

& using $\Omega_{r,0}$ & $\Omega_{m,0}$ for t_0, we get

$$\boxed{|1-\Omega(t_{rm})| \lesssim 10^{-6}}$$ ← decrease by 10^{-3}!! (even closer to 1)

Ily extrapolating further into the past:
nucleosynthesis:

$$a_{nuc} \simeq 10^{-9}$$

$$\Rightarrow |1-\Omega(t_{nuc})| \lesssim 10^{-16}$$

Q) What if we extrapolate further — till Planck time $t_p \sim 10^{-44}$ s ??

→ First, $a_p(t_p) = ?$

∵ At $t = t_p$, there's no known matter, rad. or fluid eq.,
we estimate a_p using the one observable that was present!

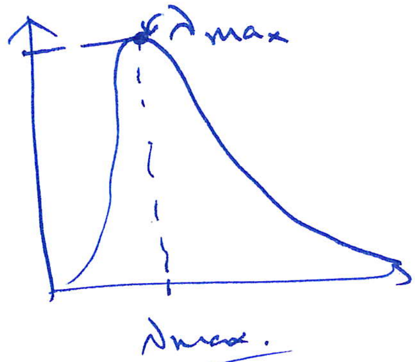
↓

Temperature!

→ Scale factor & Temperature:

From black-body rad., using Wein's law:

$$\boxed{T \propto \lambda_{max}^{-1}}$$



But, in expanding universe,

$\lambda \propto a$

$\Rightarrow \quad T \propto a^{-1}$

$$\Rightarrow \quad \boxed{\frac{T_p}{T_0} \propto \frac{a_0}{a_p}}$$

$T_0 \rightarrow$ current temp.
$T_p \rightarrow$ Temp. at t_p.

→ Using $T_0 = 2.7$ K, $T_p \sim 1.4 \times 10^{32}$ K,
& $a_0 = 1$

$$\Rightarrow \quad \boxed{a_p \approx 10^{-32}}$$

$$\Rightarrow \boxed{|1-\Omega_p| \leq 10^{-62}} \; !!$$

$\therefore$ Conclusion: The universe is fine-tuned!

→ A slight deviation from 1 at plank time or even nucleosynthesis would change the evolution of universe.

→ Can it be coincidence?

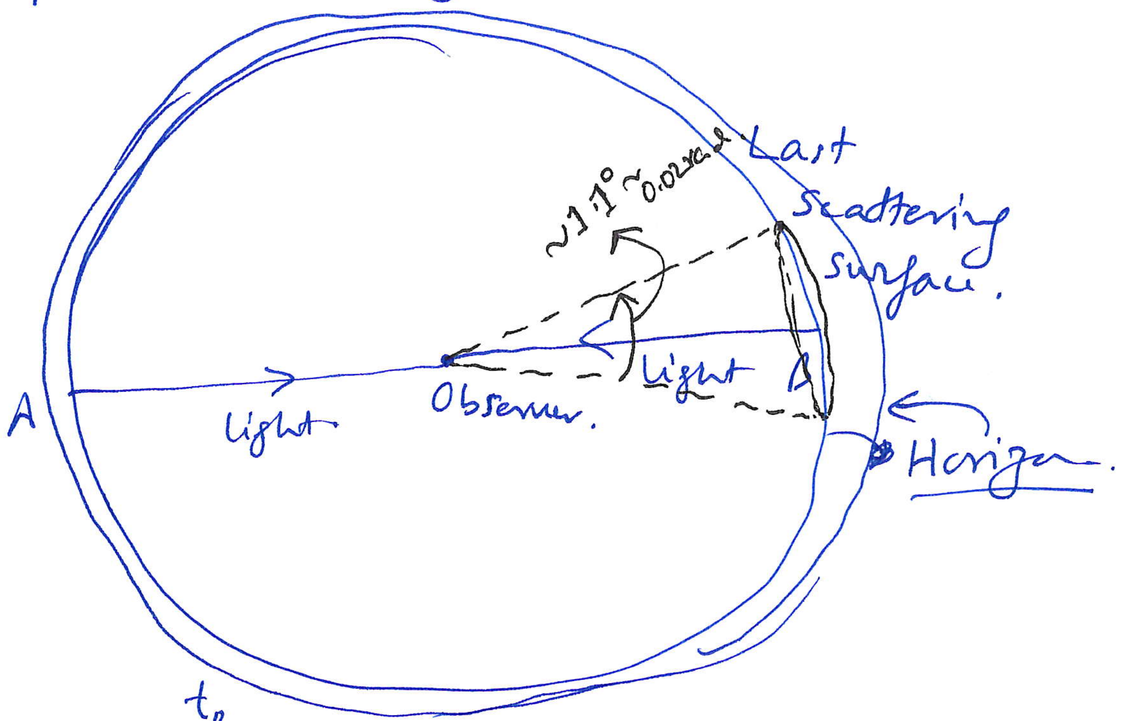
→ Scientifically unacceptable!

→ Some physical mechanism must explain why the universe so flat.

→ Flatness problem is not the only discrepancy.

(II) The Horizon problem:

"Why is the universe homogeneous and isotropic at large scales?"



$$d_{H.} = \int_{t_{ls}}^{t_0} \frac{dt}{a(t)}$$

Let, two light rays from opp. ends come to O.

→ O sees the same thing!

→ But A, B are separated by $\boxed{d_{AB} \sim 1.96\, d_{Hor}}$

⇒ A, B must be causally disconnected!

→ So how did the info about equil. reach A to B?

→ ~~If they are~~ They must be causally discon. even in the past.?

The Monopole Problem:

GUT predicts existence of magnetic monopoles near big bang.

→ But none observed in the present universe.

INFLATION

→ A single cosmological mechanism that resolves the three problems listed above!

→ Hypothesise a period in the very early universe, when universe was ~~ac~~ expanding with acceleration.

$$\boxed{\ddot{a} > 0} \longleftarrow \text{Inflation "epoch".}$$

⇒ From the acceleration eq:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) > 0$$

$$\Rightarrow \quad \rho + 3P < 0$$

$$\Rightarrow \quad \boxed{P < -\rho/3}$$

i.e EoS: $P = w\rho = -\rho \longleftarrow$ Dark energy.

⇒ Inflationary epoch was dominated by cosmological const. Λ.

With Λ, the fundamental eqs:

$$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{R_0^2 a^2} + \frac{\Lambda}{3}$$

$$\dot{\rho} = -3H(\rho + P)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) + \frac{\Lambda}{3}$$

→ Λ therefore accounts for whatever is not matter / radiation, and thus is independent of ρ, P.

∴ In ~~absence~~ a Λ dominated universe, we can neglect ρ P.

$$\Rightarrow \quad \boxed{\frac{\ddot{a}}{a} = \frac{\Lambda}{3} > 0} \longleftarrow \boxed{\Lambda > 0 !!}$$

$$\& \quad \left(\frac{\dot{a}}{a}\right)^2 = \frac{\Lambda}{3} = H^2$$

$\Rightarrow$ $H \neq H(t)$ during inflation!

Taking $\boxed{H_i = \sqrt{\frac{\Lambda}{3}}}$, & solving for $a(t)$:

$$\boxed{a(t) = a_i e^{H_i t}} \longleftarrow$$

Exponential expansion of the universe!

Q How does $a(t)$ evolve in different epochs?

$$a(t) = \begin{cases} a_i \left(t/t_i\right)^{1/2} & t < t_i \sim \text{rad. dominated.} \\ a_i e^{H_i(t-t_i)} & t_i < t < t_f \sim \text{inflationary} \\ a_i e^{H_i(t_f-t_i)} \left(t/t_f\right)^{1/2} & t > t_f \sim \text{rad. epoch.} \end{cases}$$

$\therefore$ From t_i to t_f (during inflation):

$$\boxed{\frac{a(t_f)}{a(t_i)} = e^N}$$

Where $\boxed{N \equiv H_i(t_f - t_i)}$ ← # of e-folds.

Q How does inflation solve the flatness problem?

$$|1 - \Omega(t)| = \frac{1}{R_0^2 a^2 H^2}$$

During inflation: $H \to$ const. ; $a \to a_i e^{Ht}$

$$\Rightarrow \boxed{|1-\Omega| \propto e^{-2Ht}}$$

Let, initial universe was highly curved.

i.e $|1-\Omega| \sim 1$

$\Rightarrow$ After inflation: $\boxed{|1-\Omega| \sim e^{-2N}}$

→ For sufficiently long inflation, universe will eventually become flat!!

Horizon problem soln:

$$d_{hor}\Big|_{infl.} = \int_{t_i}^{t_f} \frac{dt}{a_i\, e^{H_i(t_f - t_i)}} \propto H_i^{-1} e^{N}$$

$$\Rightarrow \boxed{d_{hor} \propto H^{-1} e^{N}}$$

Horizon size grows exponentially with inflation.

Monopole problem sol.

$$n_M \propto a^{-3} = e^{-3Ht}$$

→ n_M decays exponentially !!

⇒ No monopole today!