Gravity in the long range.

H/W Review the concept of natural units used in cosmology & high energy physics.
→ 4 fundamental units, their conversion from SI to natural units.

Ref.: Chapter 3 — B. Ryden

→ The fundamental forces in nature
* Gravity
* Electromagnetism
* Weak nuclear force
* Strong nuclear force.

Q. Which one dominates at short length scale?
— at long " " ?

→ Strong and weak nuclear forces dominate in the lengthscale $\lesssim 10^{-15}$ m.

→ The observable universe is mostly neutral ⇒ negligible electromagnetic interaction.

⇒ Gravity is the only dominant force!! (being the weakest at small lengthscale).

Q Some interesting numerical calculations:
→ what if each atom on earth would be missing an e^-?
→ ~~Between~~ two ~~e's / p^+'s~~ ~~p^+'s,~~ ~~at what distance gravity would~~

→ Gravity — Newtonian or Einsteinian?

* At least, in our solar system, ~~we~~ as per the Newtonian gravity any massive object experiences a force due to another mass, given by

$$F = -\frac{G M_g m_g}{r^2}$$

where M_g, m_g are respective masses of obj. that play a part in gravitation,

→ But, as per Newtonian mechanics,

$$F = \underbrace{m_i}_{\text{let's call this } m_i} a$$

inertial mass.

Q Is m_i the same as m_g?
→ YES!! due to Galileo!

In principle,

Given a gravitating mass M_g, an object with inertial mass m_i would move as per acceleration

$$a = -\frac{G M_g}{r^2}\left(\frac{m_g}{m_i}\right)$$

→ But we know that all objects fall to earth's surface with same acceleration!

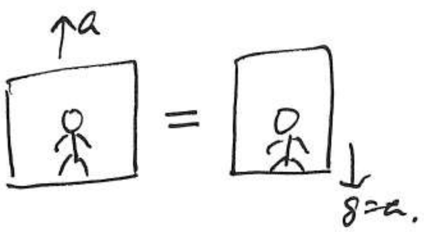
$\Rightarrow$ $\boxed{m_g = m_i}$

⇑

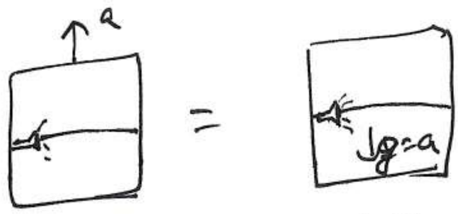
Equivalence principle.

↓

The basis for Einstein's theory of gravitation.



person



light (photons)

→ But light has no inertial mass!!

→ Also, acc. to ~~for~~ Fermat's principle in optics, ~~sp.~~ light takes the shortest route

↳ In flat space, it'd be a straight line. ↳ Eucleidean.

→ But, in presence of gravity if light is curved
⇒ the space can't be Euclidean!
⇓
A curved space(-time)?

Newton	Einstein.
* Mass tells gravity how to exert force	* Mass-energy tells spacetime how to curve.
* Force tells mass how to move.	* Curvature tells mass-energy how to move. (geodesic)

⇒ Natural explanation of equivalence principle.

Q So how do we describe this curvature?
→ Before going to STR/GTR, let's consider a 2D space.
→ We know, from trigonometry

$$\underbrace{\alpha + \beta + \gamma = \pi}_{\text{valid for a plane.}}$$

→ The distance between two infinitesimally seperated pts.

$$ds^2 = dx^2 + dy^2$$
$$= dr^2 + r^2 d\theta^2$$

Illy, if we draw Δ^{le} in sphere

$$\alpha + \beta + \gamma = \pi + A/R^2 \leftarrow \text{+ve curvature.}$$

→ On a saddle :

$$\alpha + \beta + \gamma = \pi - \frac{A}{R^2} \leftarrow \text{-ve curvature.}$$

& the distance bet. pts :

$$ds^2 = dr^2 + R^2 \sin^2(r/R)\, d\theta^2 \leftarrow \text{+ve}$$

$$ds^2 = dr^2 + R^2 \sinh^2(r/R)\, d\theta^2 \leftarrow \text{-ve.}$$

→ In 3D:

$$ds^2 = dr^2 + S_k(r)^2 \left[d\theta^2 + \sin^2\theta\, d\phi^2\right]$$

where,
$$S_k(r) = \begin{cases} R\sin(r/R) & , \; k = +1 \\ r & , \; k = 0 \\ R\sinh(r/R) & , \; k = -1 \end{cases}$$

A Brief review of STR
[Discussion > Lecture]

→ From the last lecture:
The equivalence principle implies that the geometry of "space" cannot be Euclidean!
Q Then what can it be?

→ To answer this question, we must revert to the "local" (small) lengthscale instead of large lengthscale.

→ Since it's the geometry of frames of references we are interested in, we consider free space (free of interactions).

→ Historically $\underbrace{\boxed{\text{this discussion}}}_{\text{STR}}$ precedes the $\underbrace{\boxed{\text{discussion about gravity}}}_{\text{GTR}}$.

→ We start with one of the most fundamental (earliest) principles of physics:
* The principle of relativity (by Galileo)
"No expt. can measure the absolute vel. of an observer; results of expt. are independent of the relative vel. of observer wrt others."
⇒ $\boxed{\text{Galilean geometry.}}$

→ Einstein's contribution:

"The universality of the speed of light."

i.e two unaccelerated observers will measure the same speed of light.

Q How does this "postulate" change/modify the principle of relativity?

Ex: In Galilean relativity,

relative speed of light:

$$\boxed{c' = c - v} \leftarrow \text{Not true!}$$

→ Clearly, one would have to a new (mathematical) coordinate sys.!

Define an inertial frame in "special" relativity:

→ Define an

(1) distance between two spatial pts. is independent of time.

(2) All clocks in a frame are synchronized & run at same rate.

(3) At a given time t, geometry is Euclidean.

→ Units:

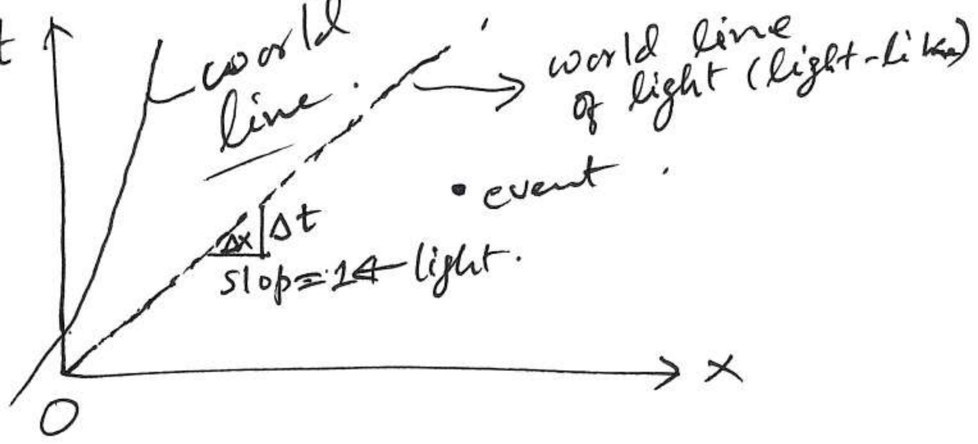
We redefine length & time SI units, so that

$$3\times10^8\ ms^{-1} = 1$$

$$\Rightarrow 1s = 3\times10^8 m$$

$$\Rightarrow \boxed{c = 1} \leftarrow \text{dimensionless \& unity.}$$

→ A world line — diag. of x-t!



→ <u>Recall</u>: The universality of light implies that the ~~speed of~~ $c = 1$ for all ~~obser~~ inertial observers $(O, O', \bar{O}, \ldots)$.
~~movin~~

In some observer's frame O:

$$\frac{\Delta x}{\Delta t} = 1$$

$$\Rightarrow \quad \Delta x = \Delta t$$

$$\Rightarrow \quad \Delta x^2 = \Delta t^2$$ ← in terms of distances

$$\Rightarrow \quad \Delta x^2 + \Delta y^2 + \Delta z^2 = \Delta t^2$$ ← Going to 3D

$$\Rightarrow \quad \boxed{\Delta x^2 + \Delta y^2 + \Delta z^2 - \Delta t^2 = 0}$$

Ilฐy in frame O':

$$\Delta x'^2 + \Delta y'^2 + \Delta z'^2 - \Delta t'^2 = 0$$

→ Here, t & t' must be different because simultaneity ~~of ti~~ is <u>not guaranteed</u> between two frames.

$\Rightarrow \Delta x'^2 + \Delta y'^2 + \Delta z'^2 - \Delta t'^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - \Delta t^2$

$\boxed{\text{H/W}}$ Read Schutz Sec. 1.6 for the proof.

$\equiv \Delta s^2$

"The invariant" of special relativity.

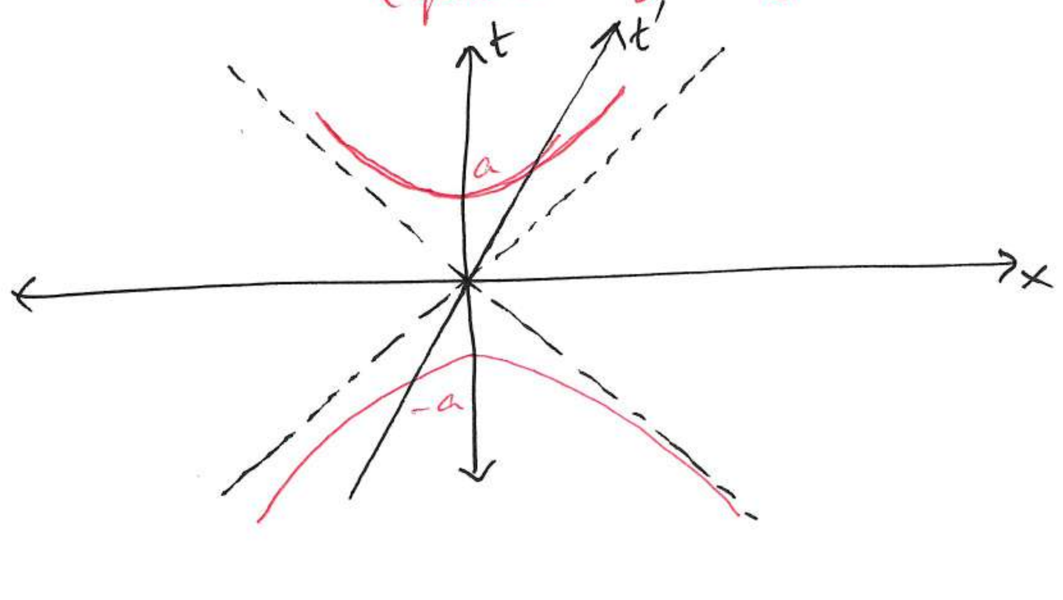
NOTE: This Δs^2 is unlike the invariant of Euclidean geometry which would have been $\Delta x_i^2 + \Delta t^2$!!

→ The space-time curve parametrized by the invariant Δs^2:

Consider 1+1 D space (& time)

i.e $\boxed{-t^2 + x^2 = a^2}$

Equation of a hyperbola.



Since, all events on a hyperbola have the same invariant parameter $\Delta s = a$,

$\rightarrow$ Consider in O:

$$\Delta x = 0, \quad \Delta t = 1$$

$$\Rightarrow \quad \Delta x^2 - \Delta t^2 = 1 = \underbrace{\Delta x'^2 - \Delta t'^2}_{\text{as measured in } O'}$$

$$\Rightarrow \quad -\Delta t^2 = -\Delta t'^2\left(1 - \left(\frac{\Delta x'}{\Delta t'}\right)^2\right)$$

$\frac{\Delta x'}{\Delta t'} \equiv v$ as measured in O'

$$\Rightarrow \quad \boxed{\Delta t' = \frac{\Delta t}{\sqrt{1 - v^2}}} \leftarrow \text{Time dilation.}$$

$\boxed{H/W}$: Do a ||er calculation when $\Delta t = 0$ & $\Delta x \neq 0$

$\downarrow$

$\boxed{\text{Length contraction}}$

$\parallel$

$$\Delta x' = \Delta x \sqrt{1 - v^2}$$

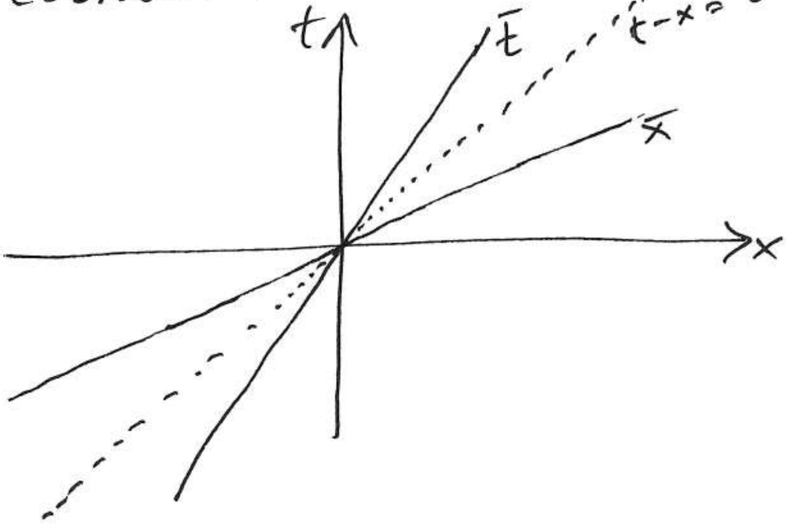
$\rightarrow$ Proper time. $\Delta \tau^2 \equiv -\Delta s^2$

$$\Rightarrow \boxed{\Delta \tau \equiv \sqrt{\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2}}$$

$\rightarrow$ Paradoxes discussion from Sec. 1.8 ~~Schwartz~~ Schutz.

The Lorentz Transformations:

→ From our geometrical understanding, the coordinate axes $\bar{t}$ & $\bar{x}$ wrt t & x:



From geometry:

Eq. of $\bar{t}$:

$$vt - x = 0 \quad \text{—— (1)}$$

$v \leq c$

Eq. of $\bar{x}$:

$$-t + vx = 0 \quad \text{—— (2)}$$

→ Attempting to construct a linear transformation:

$$\bar{t} = \alpha(t - vx) \;;\; \bar{x} = \sigma(x - vt)$$

from $\cancel{\Delta x^2 + \Delta y^2} = \Delta x^2 - \Delta t^2 = \Delta x'^2 - \Delta t'^2$

$$\Rightarrow \boxed{\bar{t} = \frac{t - vx}{\sqrt{1 - v^2}} \;;\; \bar{x} = \frac{x - vt}{\sqrt{1 - v^2}}}$$

↑ Lorentz transformation.

$\boxed{\text{H/W:}}$ what if transformation is linear?