CS365: Deep Learning

Convolutional Neural Network

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Introduction

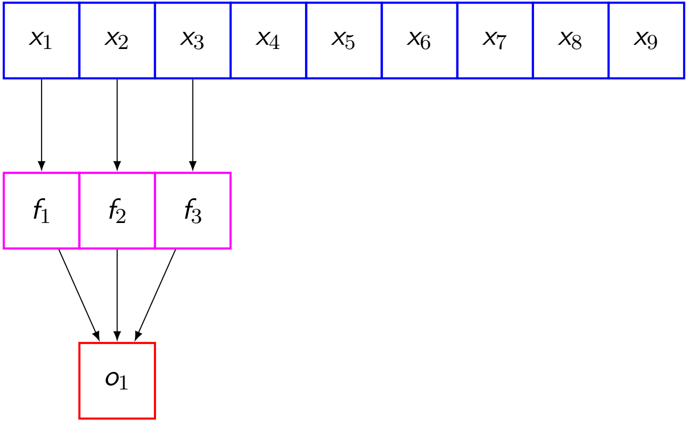
- Specialized neural network for processing data that has grid like topology
  - Time series data (one dimensional)
  - Image (two dimensional)
- Found to be reasonably suitable for certain class of problems eg. computer vision
- Instead of matrix multiplication, it uses convolution in at least one of the layers

Convolution operation

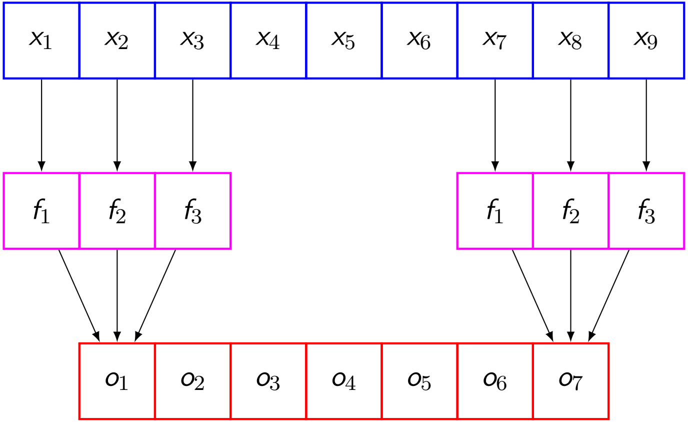
- Consider the scenario of locating a spaceship with a laser sensor
- Suppose, the sensor is noisy
  - Accurate estimation is not possible
- Weighted average of location can provide a good estimate $s(t) = \int x(a)w(t-a)da$
  - $x(a)$ — Location at age a by the sensor, t — current time, w — weight
  - This is known as convolution
  - Usually denoted as $s(t) = (x * w)(t)$
- In neural network terminology x is input, w is kernel and output is referred as feature map
- Discrete convolution can be represented as

$$s(t) = (x * w)(t) = \sum_{a=\infty}^{\infty} x(a)w(t-a)$$

Convolution in 1D



Convolution in 1D



Convolution in 2D

- In neural network input is multidimensional and so is kernel
  - These will be referred as tensor
- Two dimensional convolution can be defined as

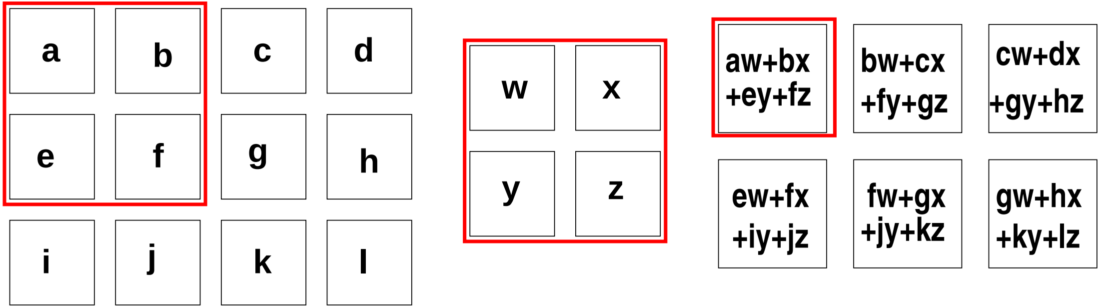
$$s(i,j) = (I * K)(i,j) = \sum_{m,n} I(m,n)k(i-m,j-n) = \sum_{m,n} I(i-m,j-n)k(m,n)$$

  - Commutative
- In many neural network, it implements as cross-correlation

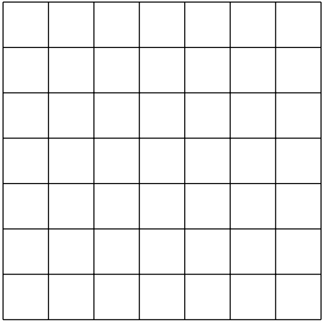
$$s(i,j) = (I * K)(i,j) = \sum_{m}\sum_{n} I(i+m,j+n)k(m,n)$$

  - No kernel flip is possible

2D convolution

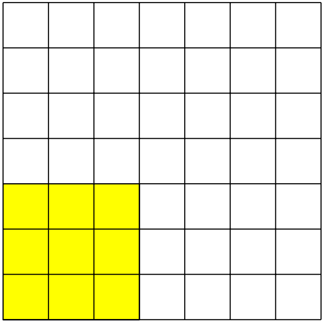


2D Convolution



Grid size: 7×7

2D Convolution

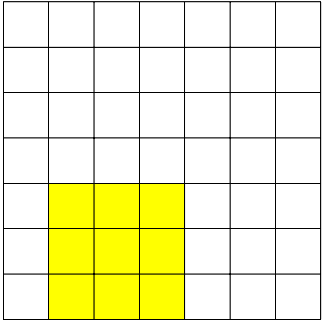


Grid size: 7×7

Filter size: 3×3

Stride: 1

2D Convolution

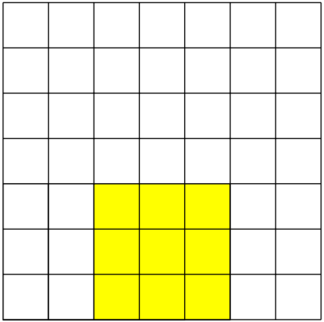


Grid size: 7×7

Filter size: 3×3

Stride: 1

2D Convolution

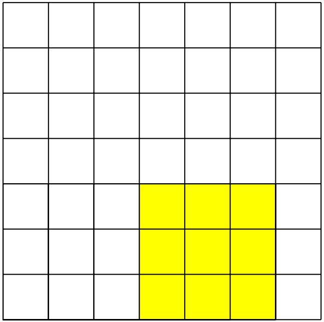


Grid size: 7×7

Filter size: 3×3

Stride: 1

2D Convolution

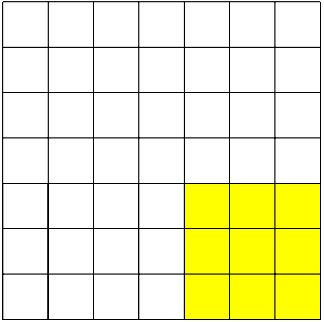


Grid size: 7×7

Filter size: 3×3

Stride: 1

2D Convolution

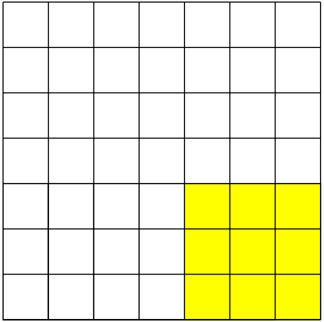


Grid size: 7×7

Filter size: 3×3

Stride: 1

2D Convolution

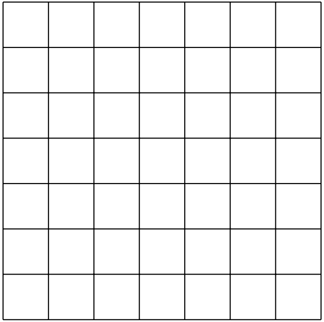


Grid size: 7×7

Filter size: 3×3
Stride: 1

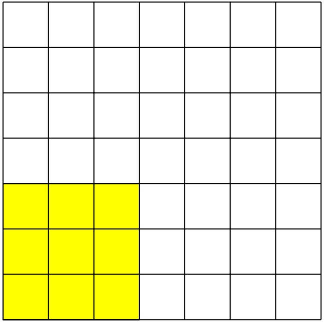
Output size: 5×5

2D convolution with stride



Grid size: 7×7

2D convolution with stride

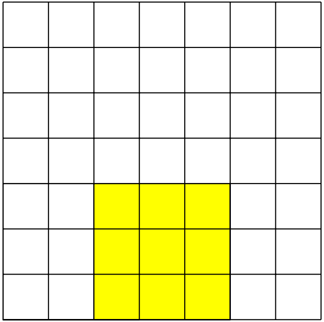


Grid size: 7×7

Filter size: 3×3

Stride: 2

2D convolution with stride

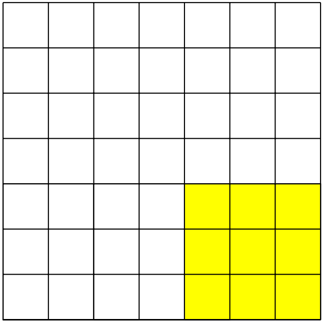


Grid size: 7×7

Filter size: 3×3

Stride: 2

2D convolution with stride

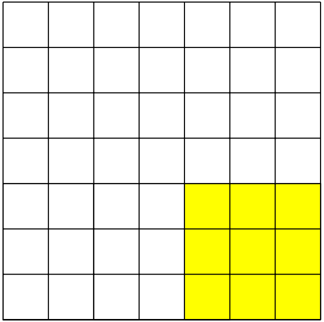


Grid size: 7×7

Filter size: 3×3

Stride: 2

2D convolution with stride



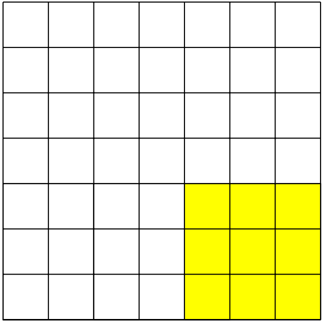
Grid size: 7×7

Filter size: 3×3

Stride: 2

Output size: 3×3

2D convolution with stride



Grid size: 7×7

Filter size: 3×3

Stride: 2

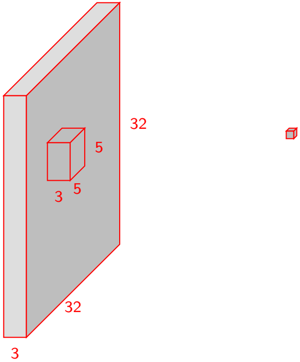
Output size: 3×3

Output size: $(N - F)/S + 1$

N - input size, F - Filter size,

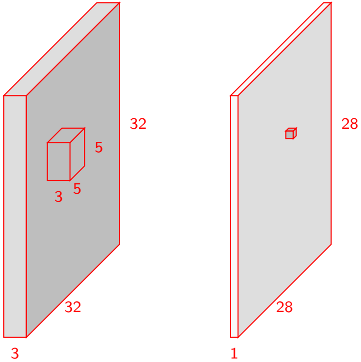
S - Stride

Convolution operation



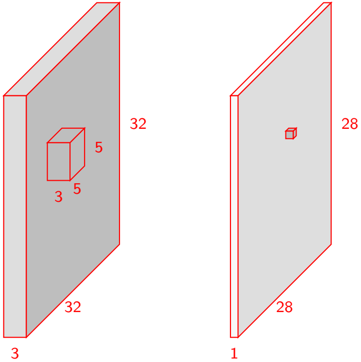
Filters are specified as 5×5. Channel depth is implicit.

Convolution operation



Filters are specified as 5×5. Channel depth is implicit.

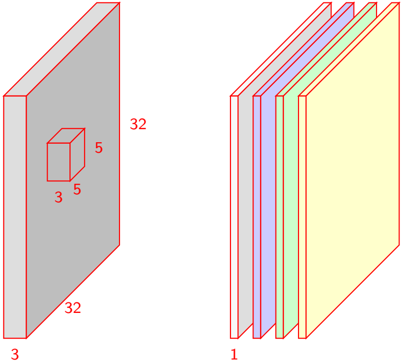
Convolution operation



Filters are specified as 5×5. Channel depth is implicit.

No. of paramters 75 excluding bias. Computation (multiplication) - $28 \times 28 \times 5 \times 5 \times 3$

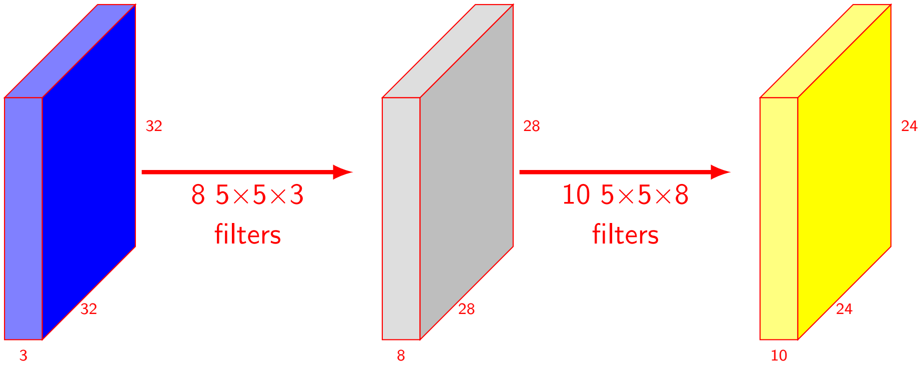
Convolution operation



Filters are specified as 5×5. Channel depth is implicit.

No. of paramters 75 excluding bias. Computation (multiplication) - $28 \times 28 \times 5 \times 5 \times 3$

Convolution example



Edge detection

- Applied filter: $[1 \quad -1]$, Original image is on the left

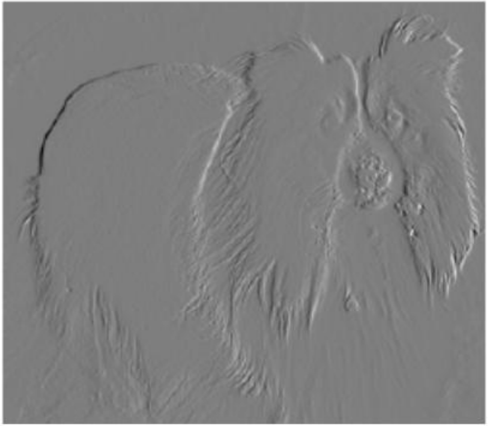


Image source: Deep Learning Book

Convolution Filter-1

1	0	-1
1	0	-1
1	0	-1

Convolution Filter-1

1	1	1	0	0	0
1	1	1	0	0	0
1	1	1	0	0	0
1	1	1	0	0	0
1	1	1	0	0	0
1	1	1	0	0	0

$\otimes$

1	0	-1
1	0	-1
1	0	-1

$=$

Convolution Filter-1

$$
\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 3 & 3 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 3 & 3 & 0 \end{bmatrix}
$$

Convolution Filter-2

1	1	1
0	0	0
-1	-1	-1

Convolution Filter-2

$$\begin{array}{|c|c|c|c|c|c|}\hline 1 & 1 & 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 & 1 & 1 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ \hline \end{array} \otimes \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 0 & 0 & 0 \\ \hline -1 & -1 & -1 \\ \hline \end{array} =$$

Convolution Filter-2

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

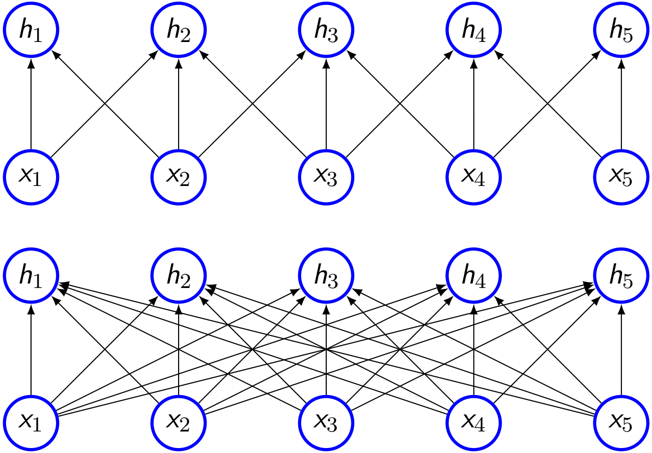
Advantages

- Convolution can exploit the following properties
  - Sparse interaction (Also known as sparse connectivity or sparse weights)
  - Parameter sharing
  - Equivariant representation

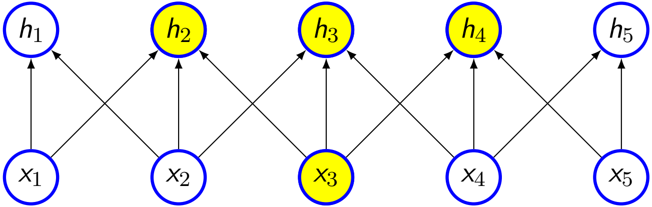
Sparse interaction

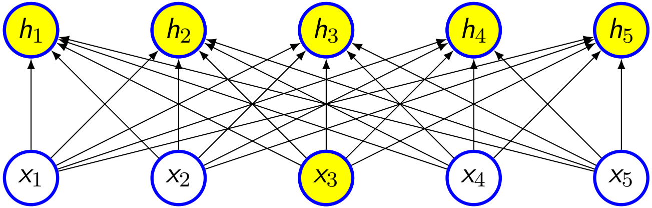
- Traditional neural network layers use matrix multiplication to describe how outputs and inputs are related
- Convolution uses a smaller kernel
  - Significant reduction in number of parameters
  - Computing output require few comparison
- For example, if there is m inputs and n outputs, traditional neural network will require $m \times n$ parameters
- If each of the output is connected to at most k units, the number of parameters will be $k \times n$

Sparse connectivity

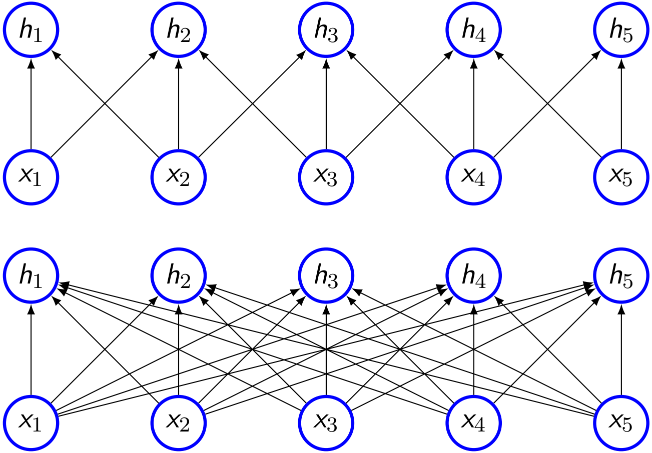


Sparse connectivity

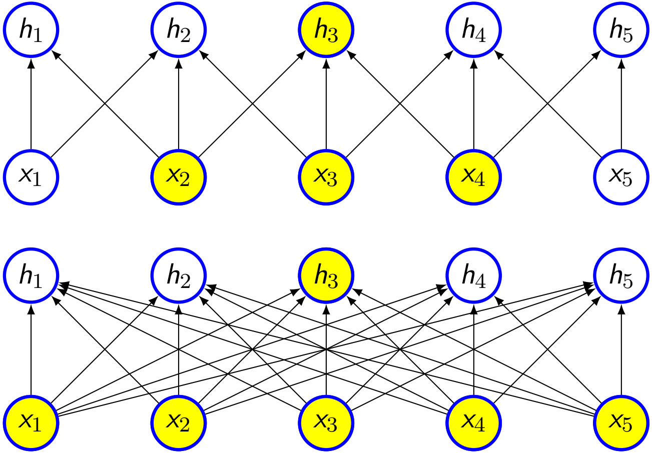




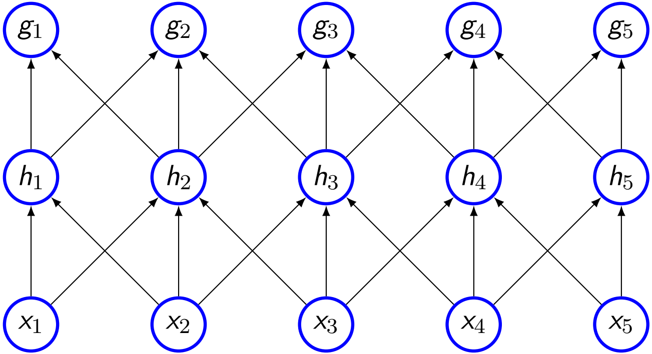
Sparse connectivity



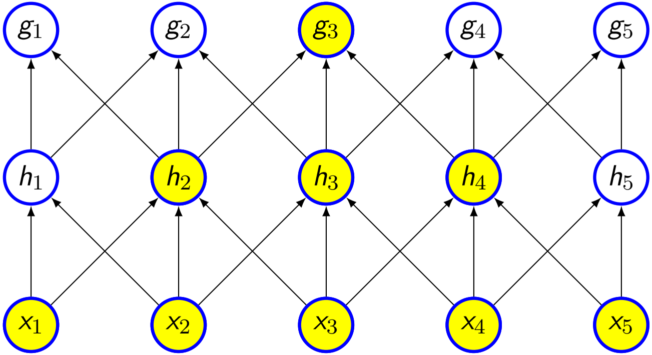
Sparse connectivity



Receptive field



Receptive field



Parameter sharing

- Same parameters are used for more than one function model
- In tradition neural network, weight is used only once
- Each member of kernel is used at every position of the inputs
- As $k \ll m$, the number of parameters will reduced significantly
- Also, require less memory

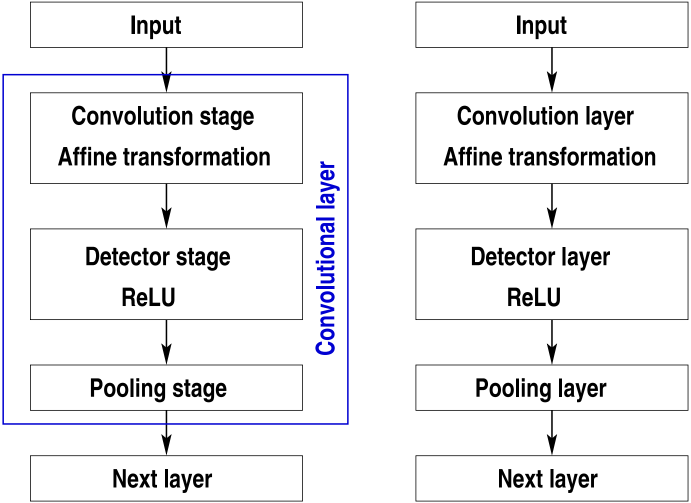
Equivariance

- If the input changes, the output changes in the same way
- Specifically, a function $f(x)$ is equivariant to function g if $f(g(x)) = g(f(x))$
  - Example, g is a linear translation
  - Let B be a function giving image brightness at some integer coordinates and g be a function mapping from one image to another image function such that $I' = g(I)$ with $I'(x, y) = I(x - 1, y)$
- There are cases sharing of parameters across the entire image is not a good idea

Pooling

- Typical convolutional network has three stages
  - **Convolution** — several convolution to produce linear activation
  - **Detector stage** — linear activation runs through the non-linear unit such as ReLU
  - **Pooling** — Output is updated with a summary of statistics of nearby inputs
    - Maxpooling reports the maximum output within a rectangular neighbourhood
    - Average of rectangular neighbourhood
    - Weighted average using central pixel
- Pooling helps to make representation invariant to small translation
  - Feature is more important than where it is present
- Pooling helps in case of variable size of inputs

Typical CNN



Input
Convolution stage
Affine transformation
Detector stage
ReLU
Pooling stage
Convolutional layer
Next layer
Input
Convolution layer
Affine transformation
Detector layer
ReLU
Pooling layer
Next layer

Max Pool

0	4	7	8
9	2	4	5
6	7	3	4
8	2	1	5

Max Pool

0	4	7	8
9	2	4	5
6	7	3	4
8	2	1	5

8	

Max Pool

0	4	7	8
9	2	4	5
6	7	3	4
8	2	1	5

8	5

Max Pool

0	4	7	8
9	2	4	5
6	7	3	4
8	2	1	5

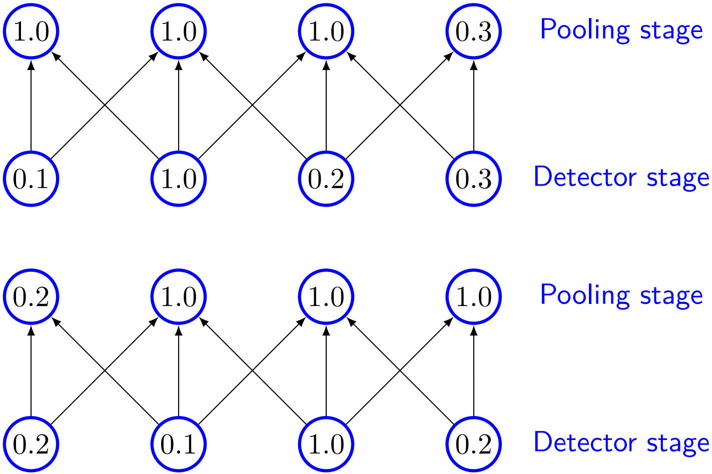
9	
8	5

Max Pool

0	4	7	8
9	2	4	5
6	7	3	4
8	2	1	5

9	8
8	5

Invariance of maxpooling



Learned invariances

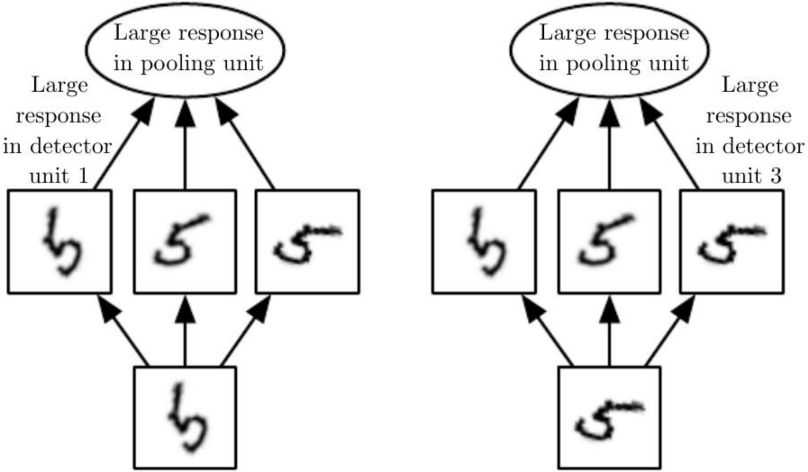
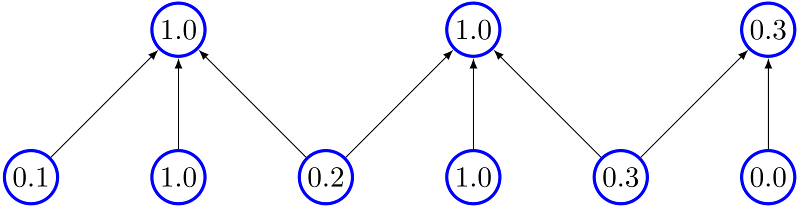
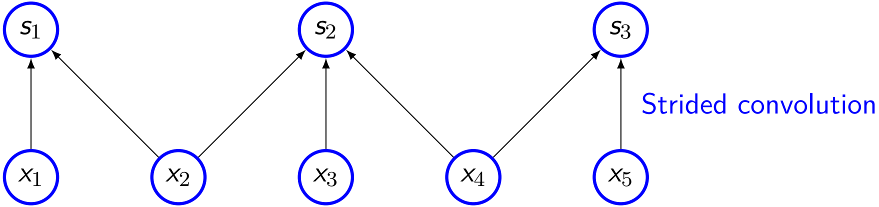


Image source: Deep Learning Book

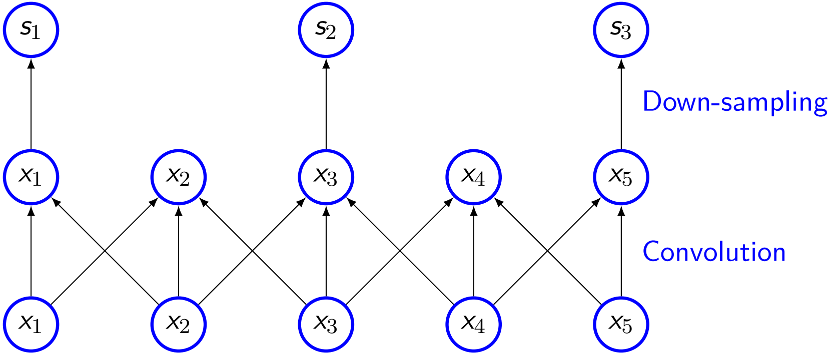
Pooling with downsampling



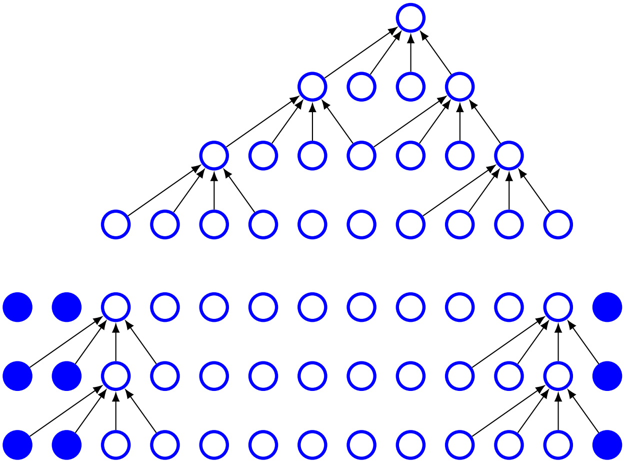
Strided convolution



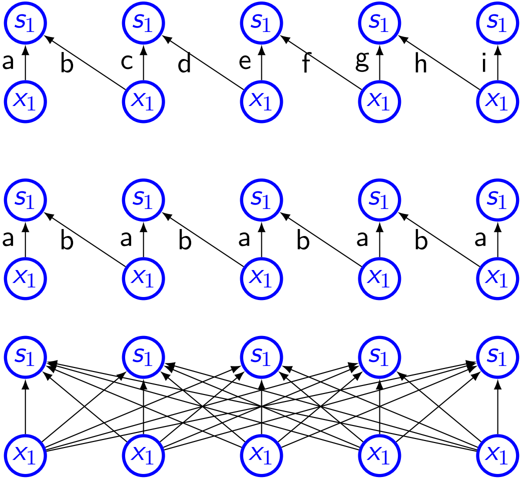
Strided convolution (contd)



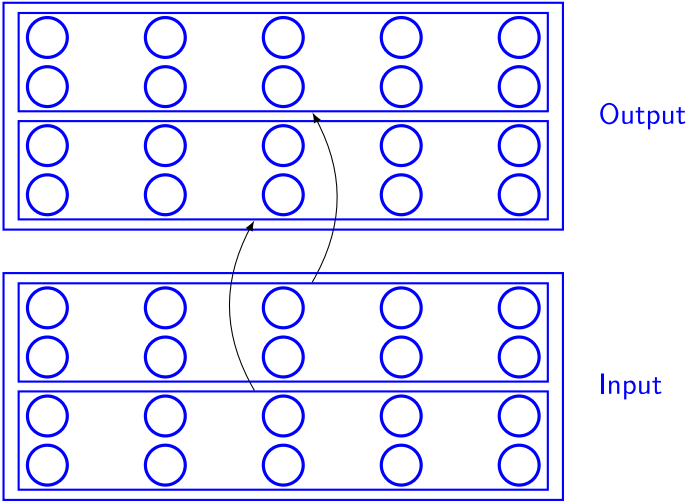
Zero padding



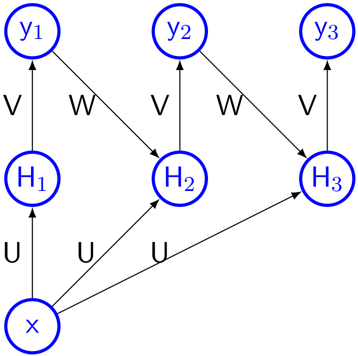
Connections



Local convolution



Recurrent convolution network



CNN & Backpropagation-1

x_{11}	x_{12}	x_{13}
x_{21}	x_{22}	x_{23}
x_{31}	x_{32}	x_{33}

f_{11}	f_{12}
f_{21}	f_{22}

CNN & Backpropagation-1

x_{11}	x_{12}	x_{13}
x_{21}	x_{22}	x_{23}
x_{31}	x_{32}	x_{33}

$x_{11} f_{11}$	$x_{12} f_{12}$	x_{13}
$x_{21} f_{21}$	$x_{22} f_{22}$	x_{23}
x_{31}	x_{32}	x_{33}

f_{11}	f_{12}
f_{21}	f_{22}

CNN & Backpropagation-1

x_{11}	x_{12}	x_{13}
x_{21}	x_{22}	x_{23}
x_{31}	x_{32}	x_{33}

x_{11}	x_{12}	x_{13}
x_{21}	$x_{22} f_{11}$	$x_{23} f_{12}$
x_{31}	$x_{32} f_{21}$	$x_{33} f_{22}$

f_{11}	f_{12}
f_{21}	f_{22}

CNN & Backpropagation-1

x_{11}	x_{12}	x_{13}
x_{21}	x_{22}	x_{23}
x_{31}	x_{32}	x_{33}

x_{11}	x_{12}	x_{13}
x_{21}	$x_{22}f_{11}$	$x_{23}f_{12}$
x_{31}	$x_{32}f_{21}$	$x_{33}f_{22}$

f_{11}	f_{12}
f_{21}	f_{22}

$$
\begin{aligned}
o_{11} &= x_{11}f_{11} + x_{12}f_{12} + x_{21}f_{21} + x_{22}f_{22} \\
o_{12} &= x_{12}f_{11} + x_{13}f_{12} + x_{22}f_{21} + x_{23}f_{22} \\
o_{21} &= x_{21}f_{11} + x_{22}f_{12} + x_{31}f_{21} + x_{32}f_{22} \\
o_{22} &= x_{22}f_{11} + x_{23}f_{12} + x_{32}f_{21} + x_{33}f_{22}
\end{aligned}
$$

CNN & Backpropagation-2

- Gradient with respect to filter: $\dfrac{\partial L}{\partial F} = \dfrac{\partial L}{\partial o} \times \dfrac{\partial o}{\partial F}$
- Gradient with respect to every element: $\dfrac{\partial L}{\partial F_i} = \sum_{k=1}^{m} \dfrac{\partial L}{\partial o_k} \times \dfrac{\partial o_k}{\partial F_i}$

CNN & Backpropagation-2

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- Hence,

$$\frac{\partial L}{\partial F_{11}} =$$

CNN & Backpropagation-2

- Gradient with respect to filter: $\dfrac{\partial L}{\partial F} = \dfrac{\partial L}{\partial o} \times \dfrac{\partial o}{\partial F}$
- Gradient with respect to every element: $\dfrac{\partial L}{\partial F_i} = \sum_{k=1}^{m} \dfrac{\partial L}{\partial o_k} \times \dfrac{\partial o_k}{\partial F_i}$
- Hence,

$$\frac{\partial L}{\partial F_{11}} = \frac{\partial L}{\partial o_{11}} \times \frac{\partial o_{11}}{\partial F_{11}} + \frac{\partial L}{\partial o_{12}} \times \frac{\partial o_{12}}{\partial F_{11}} + \frac{\partial L}{\partial o_{21}} \times \frac{\partial o_{21}}{\partial F_{11}} + \frac{\partial L}{\partial o_{22}} \times \frac{\partial o_{22}}{\partial F_{11}}$$

CNN & Backpropagation-2

- Gradient with respect to filter: $\frac{\partial L}{\partial F} = \frac{\partial L}{\partial o} \times \frac{\partial o}{\partial F}$
- Gradient with respect to every element: $\frac{\partial L}{\partial F_i} = \sum_{k=1}^{m} \frac{\partial L}{\partial o_k} \times \frac{\partial o_k}{\partial F_i}$
- Hence,

$$\frac{\partial L}{\partial F_{11}} = \frac{\partial L}{\partial o_{11}} \times \frac{\partial o_{11}}{\partial F_{11}} + \frac{\partial L}{\partial o_{12}} \times \frac{\partial o_{12}}{\partial F_{11}} + \frac{\partial L}{\partial o_{21}} \times \frac{\partial o_{21}}{\partial F_{11}} + \frac{\partial L}{\partial o_{22}} \times \frac{\partial o_{22}}{\partial F_{11}}$$

$$\frac{\partial L}{\partial F_{12}} = \frac{\partial L}{\partial o_{11}} \times \frac{\partial o_{11}}{\partial F_{12}} + \frac{\partial L}{\partial o_{12}} \times \frac{\partial o_{12}}{\partial F_{12}} + \frac{\partial L}{\partial o_{21}} \times \frac{\partial o_{21}}{\partial F_{12}} + \frac{\partial L}{\partial o_{22}} \times \frac{\partial o_{22}}{\partial F_{12}}$$

$$\frac{\partial L}{\partial F_{21}} = \frac{\partial L}{\partial o_{11}} \times \frac{\partial o_{11}}{\partial F_{21}} + \frac{\partial L}{\partial o_{12}} \times \frac{\partial o_{12}}{\partial F_{21}} + \frac{\partial L}{\partial o_{21}} \times \frac{\partial o_{21}}{\partial F_{21}} + \frac{\partial L}{\partial o_{22}} \times \frac{\partial o_{22}}{\partial F_{21}}$$

$$\frac{\partial L}{\partial F_{22}} = \frac{\partial L}{\partial o_{11}} \times \frac{\partial o_{11}}{\partial F_{22}} + \frac{\partial L}{\partial o_{12}} \times \frac{\partial o_{12}}{\partial F_{22}} + \frac{\partial L}{\partial o_{21}} \times \frac{\partial o_{21}}{\partial F_{22}} + \frac{\partial L}{\partial o_{22}} \times \frac{\partial o_{22}}{\partial F_{22}}$$

CNN & Backpropagation-3

- After simplification, we get

$$\frac{\partial L}{\partial F_{11}} = \frac{\partial L}{\partial o_{11}} \times x_{11} + \frac{\partial L}{\partial o_{12}} \times x_{12} + \frac{\partial L}{\partial o_{21}} \times x_{21} + \frac{\partial L}{\partial o_{22}} \times x_{22}$$

$$\frac{\partial L}{\partial F_{12}} = \frac{\partial L}{\partial o_{11}} \times x_{12} + \frac{\partial L}{\partial o_{12}} \times x_{13} + \frac{\partial L}{\partial o_{21}} \times x_{22} + \frac{\partial L}{\partial o_{22}} \times x_{23}$$

$$\frac{\partial L}{\partial F_{21}} = \frac{\partial L}{\partial o_{11}} \times x_{21} + \frac{\partial L}{\partial o_{12}} \times x_{22} + \frac{\partial L}{\partial o_{21}} \times x_{31} + \frac{\partial L}{\partial o_{22}} \times x_{32}$$

$$\frac{\partial L}{\partial F_{22}} = \frac{\partial L}{\partial o_{11}} \times x_{22} + \frac{\partial L}{\partial o_{12}} \times x_{23} + \frac{\partial L}{\partial o_{21}} \times x_{32} + \frac{\partial L}{\partial o_{22}} \times x_{33}$$

CNN & Backpropagation-3

- After simplification, we get

$$\frac{\partial L}{\partial F_{11}} = \frac{\partial L}{\partial o_{11}} \times x_{11} + \frac{\partial L}{\partial o_{12}} \times x_{12} + \frac{\partial L}{\partial o_{21}} \times x_{21} + \frac{\partial L}{\partial o_{22}} \times x_{22}$$

$$\frac{\partial L}{\partial F_{12}} = \frac{\partial L}{\partial o_{11}} \times x_{12} + \frac{\partial L}{\partial o_{12}} \times x_{13} + \frac{\partial L}{\partial o_{21}} \times x_{22} + \frac{\partial L}{\partial o_{22}} \times x_{23}$$

$$\frac{\partial L}{\partial F_{21}} = \frac{\partial L}{\partial o_{11}} \times x_{21} + \frac{\partial L}{\partial o_{12}} \times x_{22} + \frac{\partial L}{\partial o_{21}} \times x_{31} + \frac{\partial L}{\partial o_{22}} \times x_{32}$$

$$\frac{\partial L}{\partial F_{22}} = \frac{\partial L}{\partial o_{11}} \times x_{22} + \frac{\partial L}{\partial o_{12}} \times x_{23} + \frac{\partial L}{\partial o_{21}} \times x_{32} + \frac{\partial L}{\partial o_{22}} \times x_{33}$$

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} \otimes \begin{bmatrix} \frac{\partial L}{\partial o_{11}} & \frac{\partial L}{\partial o_{12}} \\ \frac{\partial L}{\partial o_{21}} & \frac{\partial L}{\partial o_{22}} \end{bmatrix} = \begin{bmatrix} \frac{\partial L}{\partial f_{11}} & \frac{\partial L}{\partial f_{12}} \\ \frac{\partial L}{\partial f_{21}} & \frac{\partial L}{\partial f_{22}} \end{bmatrix}$$

CNN & Backpropagation-4

- Gradient with respect to filter: $\dfrac{\partial L}{\partial x} = \dfrac{\partial L}{\partial o} \times \dfrac{\partial o}{\partial x}$
- Gradient with respect to every element: $\dfrac{\partial L}{\partial x_i} = \sum_{k=1}^{m} \dfrac{\partial L}{\partial o_k} \times \dfrac{\partial o_k}{\partial x_i}$

CNN & Backpropagation-4

- Gradient with respect to filter: $\frac{\partial L}{\partial x} = \frac{\partial L}{\partial o} \times \frac{\partial o}{\partial x}$
- Gradient with respect to every element: $\frac{\partial L}{\partial x_i} = \sum_{k=1}^{m} \frac{\partial L}{\partial o_k} \times \frac{\partial o_k}{\partial x_i}$
- Hence,

  $\frac{\partial L}{\partial x_{11}} =$

CNN & Backpropagation-4

- Gradient with respect to filter: $\dfrac{\partial L}{\partial x} = \dfrac{\partial L}{\partial o} \times \dfrac{\partial o}{\partial x}$
- Gradient with respect to every element: $\dfrac{\partial L}{\partial x_i} = \sum_{k=1}^{m} \dfrac{\partial L}{\partial o_k} \times \dfrac{\partial o_k}{\partial x_i}$
- Hence,

$\frac{\partial L}{\partial x_{11}} = \frac{\partial L}{\partial o_{11}} \times f_{11}$ $\qquad \frac{\partial L}{\partial x_{12}} =$

Deep Learning

CNN & Backpropagation-4

- Gradient with respect to filter: $\dfrac{\partial L}{\partial x} = \dfrac{\partial L}{\partial o} \times \dfrac{\partial o}{\partial x}$
- Gradient with respect to every element: $\dfrac{\partial L}{\partial x_i} = \sum_{k=1}^{m} \dfrac{\partial L}{\partial o_k} \times \dfrac{\partial o_k}{\partial x_i}$
- Hence,

$$\frac{\partial L}{\partial x_{11}} = \frac{\partial L}{\partial o_{11}} \times f_{11} \qquad \frac{\partial L}{\partial x_{12}} = \frac{\partial L}{\partial o_{11}} \times f_{12} + \frac{\partial L}{\partial o_{12}} \times f_{11}$$

CNN & Backpropagation-4

- Gradient with respect to filter: $\dfrac{\partial L}{\partial x} = \dfrac{\partial L}{\partial o} \times \dfrac{\partial o}{\partial x}$
- Gradient with respect to every element: $\dfrac{\partial L}{\partial x_i} = \sum_{k=1}^{m} \dfrac{\partial L}{\partial o_k} \times \dfrac{\partial o_k}{\partial x_i}$
- Hence,

$$\frac{\partial L}{\partial x_{11}} = \frac{\partial L}{\partial o_{11}} \times f_{11} \qquad \frac{\partial L}{\partial x_{12}} = \frac{\partial L}{\partial o_{11}} \times f_{12} + \frac{\partial L}{\partial o_{12}} \times f_{11}$$

$$\frac{\partial L}{\partial x_{13}} = \frac{\partial L}{\partial o_{12}} \times f_{12} \qquad \frac{\partial L}{\partial x_{21}} = \frac{\partial L}{\partial o_{11}} \times f_{21} + \frac{\partial L}{\partial o_{21}} \times f_{11}$$

$$\frac{\partial L}{\partial x_{22}} =$$

CNN & Backpropagation-4

- Gradient with respect to filter: $\dfrac{\partial L}{\partial x} = \dfrac{\partial L}{\partial o} \times \dfrac{\partial o}{\partial x}$
- Gradient with respect to every element: $\dfrac{\partial L}{\partial x_i} = \sum_{k=1}^{m} \dfrac{\partial L}{\partial o_k} \times \dfrac{\partial o_k}{\partial x_i}$
- Hence,

$$\frac{\partial L}{\partial x_{11}} = \frac{\partial L}{\partial o_{11}} \times f_{11} \qquad \frac{\partial L}{\partial x_{12}} = \frac{\partial L}{\partial o_{11}} \times f_{12} + \frac{\partial L}{\partial o_{12}} \times f_{11}$$

$$\frac{\partial L}{\partial x_{13}} = \frac{\partial L}{\partial o_{12}} \times f_{12} \qquad \frac{\partial L}{\partial x_{21}} = \frac{\partial L}{\partial o_{11}} \times f_{21} + \frac{\partial L}{\partial o_{21}} \times f_{11}$$

$$\frac{\partial L}{\partial x_{22}} = \frac{\partial L}{\partial o_{12}} \times f_{22} + \frac{\partial L}{\partial x_{12}} \times f_{21} + \frac{\partial L}{\partial o_{21}} \times f_{12} + \frac{\partial L}{\partial o_{22}} \times f_{11}$$

CNN & Backpropagation-4

- Gradient with respect to filter: $\frac{\partial L}{\partial x} = \frac{\partial L}{\partial o} \times \frac{\partial o}{\partial x}$
- Gradient with respect to every element: $\frac{\partial L}{\partial x_i} = \sum_{k=1}^{m} \frac{\partial L}{\partial o_k} \times \frac{\partial o_k}{\partial x_i}$
- Hence,

$$\frac{\partial L}{\partial x_{11}} = \frac{\partial L}{\partial o_{11}} \times f_{11} \qquad \frac{\partial L}{\partial x_{12}} = \frac{\partial L}{\partial o_{11}} \times f_{12} + \frac{\partial L}{\partial o_{12}} \times f_{11}$$

$$\frac{\partial L}{\partial x_{13}} = \frac{\partial L}{\partial o_{12}} \times f_{12} \qquad \frac{\partial L}{\partial x_{21}} = \frac{\partial L}{\partial o_{11}} \times f_{21} + \frac{\partial L}{\partial o_{21}} \times f_{11}$$

$$\frac{\partial L}{\partial x_{22}} = \frac{\partial L}{\partial o_{12}} \times f_{22} + \frac{\partial L}{\partial x_{12}} \times f_{21} + \frac{\partial L}{\partial o_{21}} \times f_{12} + \frac{\partial L}{\partial o_{22}} \times f_{11}$$

$$\frac{\partial L}{\partial x_{23}} = \frac{\partial L}{\partial o_{12}} \times f_{22} + \frac{\partial L}{\partial o_{22}} \times f_{12} \qquad \frac{\partial L}{\partial x_{31}} = \frac{\partial L}{\partial o_{21}} \times f_{21}$$

$$\frac{\partial L}{\partial x_{32}} = \frac{\partial L}{\partial o_{21}} \times f_{22} + \frac{\partial L}{\partial o_{22}} \times f_{21} \qquad \frac{\partial L}{\partial x_{33}} = \frac{\partial L}{\partial o_{22}} \times f_{22}$$

CNN & Backpropagation-5

$$
\begin{bmatrix}
\frac{\partial L}{\partial x_{11}} & \frac{\partial L}{\partial x_{12}} & \frac{\partial L}{\partial x_{13}} \\
\frac{\partial L}{\partial x_{21}} & \frac{\partial L}{\partial x_{22}} & \frac{\partial L}{\partial x_{23}} \\
\frac{\partial L}{\partial x_{31}} & \frac{\partial L}{\partial x_{32}} & \frac{\partial L}{\partial x_{33}}
\end{bmatrix}
=
\begin{bmatrix}
f_{22} & f_{21} \\
f_{12} & f_{11}
\end{bmatrix}
\otimes
\begin{bmatrix}
\frac{\partial L}{\partial o_{11}} & \frac{\partial L}{\partial o_{12}} \\
\frac{\partial L}{\partial o_{21}} & \frac{\partial L}{\partial o_{22}}
\end{bmatrix}
$$

CNN Architectures

- AlexNet
- VggNet
- GoogleNet
- ResNet
- SENet
- Wide ResNet
- ResNeXT
- DenseNet
- MobileNets
- NASNet
- EfficientNet
- ... and many more

ImageNet Challenge

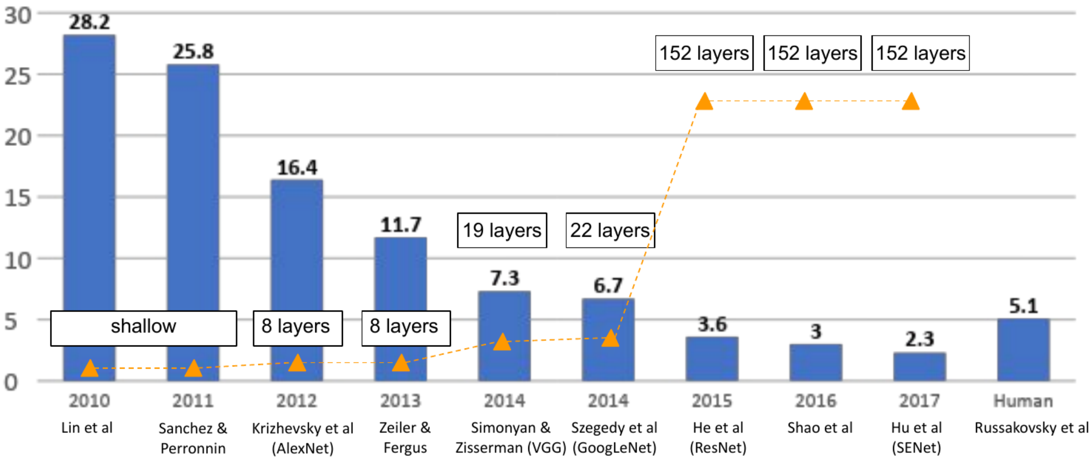


Image source: Jiajun et al. CS230 slides

AlexNet

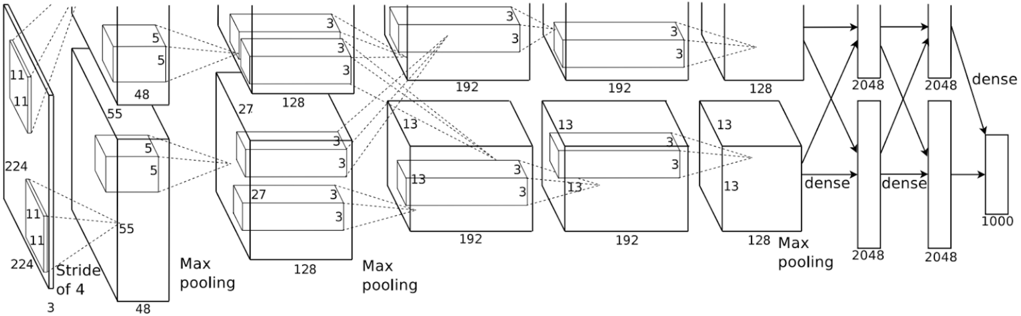
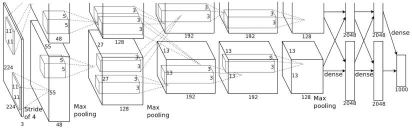


Image source: https://worksheets.codalab.org

AlexNet



- Architecture

  - INPUT - $227 \times 227 \times 3$
  - CONV1 - 96 11×11 filters at stride 4, pad 0, Output: $55 \times 55 \times 96$
  - MAX POOL1 - 3×3 filter, stride 2 Output: $27 \times 27 \times 96$
  - NORM1 - Output: $27 \times 27 \times 96$
  - CONV2 - 256 5×5 filters at stride 1, pad 2, Output: $27 \times 27 \times 256$
  - MAX POOL2 - 3×3 filter, stride 2 Output: $13 \times 13 \times 256$
  - NORM2 - O $13 \times 13 \times 256$
  - CONV3 - 384 3×3 filter, stride 1, pad 1, Output: $13 \times 13 \times 384$
  - CONV4 - 384 3×3 filter, stride 1, pad 1, Output: $13 \times 13 \times 384$
  - CONV5 - 256 3×3 filter, stride 1, pad 1, Output: O $13 \times 13 \times 256$
  - MAX POOL3 - 3×3 filter, stride 2, Output: $6 \times 6 \times 256$
  - FC6 - 4096 Neurons
  - FC7 - 4096 Neurons
  - FC8 - 1000 Neurons

Image source: https://worksheets.codalab.org

ZFNet

- Almost similar to AlexNet
- CONV1: changes from (11x11 stride 4) to (7x7 stride 2)
- CONV3,4,5: instead of 384, 384, 256 filters use 512, 1024, 512
- Error reduces to 11.7% from 16.4% (top 5)

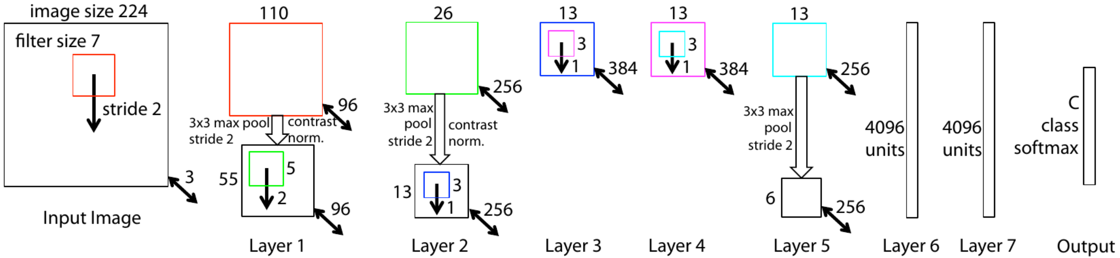


Image source: Visualizing and Understanding Convolutional Networks

VggNet: VGG16

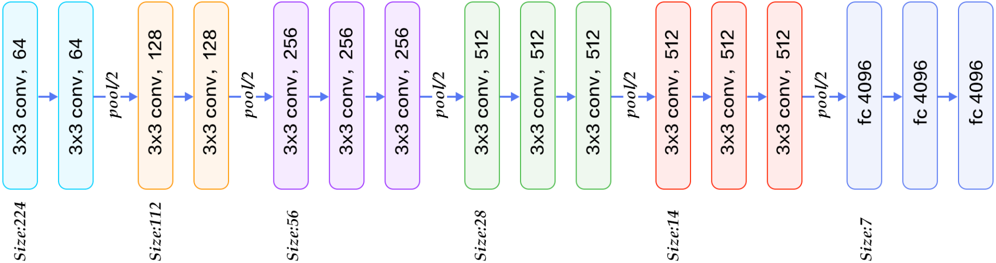


Image source: internet

VggNet: Vgg16 vs Vgg19

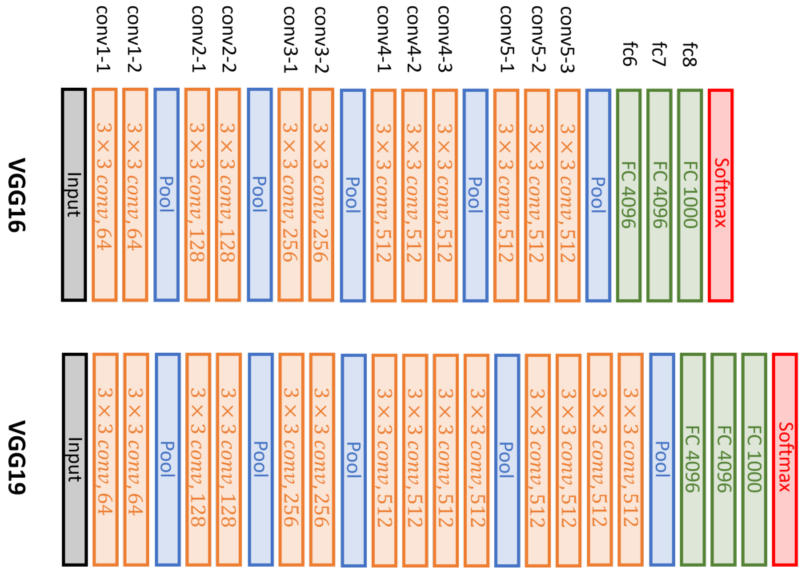


Image source: internet

Convolution Filter-2

Layer	Size	Memory	Params
Input	224x224x3	150k	0

Convolution Filter-2

Layer	Size	Memory	Params
Input	224x224x3	150k	0
C3-64	224x224x64		

Convolution Filter-2

Layer	Size	Memory	Params
Input	224x224x3	150k	0
C3-64	224x224x64	3.2M	1728
C3-64	224x224x64		

Convolution Filter-2

Layer	Size	Memory	Params
Input	224x224x3	150k	0
C3-64	224x224x64	3.2M	1728
C3-64	224x224x64	3.2M	36864
Pool2	112x112x64		

Convolution Filter-2

Layer	Size	Memory	Params
Input	224x224x3	150k	0
C3-64	224x224x64	3.2M	1728
C3-64	224x224x64	3.2M	36864
Pool2	112x112x64	800k	0
C3-128	112x112x128		

Convolution Filter-2

Layer	Size	Memory	Params
Input	224x224x3	150k	0
C3-64	224x224x64	3.2M	1728
C3-64	224x224x64	3.2M	36864
Pool2	112x112x64	800k	0
C3-128	112x112x128	1.6M	73728
C3-128	112x112x128		

Convolution Filter-2

Layer	Size	Memory	Params
Input	224x224x3	150k	0
C3-64	224x224x64	3.2M	1728
C3-64	224x224x64	3.2M	36864
Pool2	112x112x64	800k	0
C3-128	112x112x128	1.6M	73728
C3-128	112x112x128	1.6M	147456
Pool2	56x56x128		

Convolution Filter-2

Layer	Size	Memory	Params
Input	224x224x3	150k	0
C3-64	224x224x64	3.2M	1728
C3-64	224x224x64	3.2M	36864
Pool2	112x112x64	800k	0
C3-128	112x112x128	1.6M	73728
C3-128	112x112x128	1.6M	147456
Pool2	56x56x128	400k	0
C3-256	56x56x256		

Convolution Filter-2

Layer	Size	Memory	Params
Input	224x224x3	150k	0
C3-64	224x224x64	3.2M	1728
C3-64	224x224x64	3.2M	36864
Pool2	112x112x64	800k	0
C3-128	112x112x128	1.6M	73728
C3-128	112x112x128	1.6M	147456
Pool2	56x56x128	400k	0
C3-256	56x56x256	800k	294912
C3-256	56x56x256		

Convolution Filter-2

Layer	Size	Memory	Params
Input	224x224x3	150k	0
C3-64	224x224x64	3.2M	1728
C3-64	224x224x64	3.2M	36864
Pool2	112x112x64	800k	0
C3-128	112x112x128	1.6M	73728
C3-128	112x112x128	1.6M	147456
Pool2	56x56x128	400k	0
C3-256	56x56x256	800k	294912
C3-256	56x56x256	800k	589824
C3-256	56x56x256	800k	589824
Pool2	28x28x256		

Convolution Filter-2

Layer	Size	Memory	Params
Input	224x224x3	150k	0
C3-64	224x224x64	3.2M	1728
C3-64	224x224x64	3.2M	36864
Pool2	112x112x64	800k	0
C3-128	112x112x128	1.6M	73728
C3-128	112x112x128	1.6M	147456
Pool2	56x56x128	400k	0
C3-256	56x56x256	800k	294912
C3-256	56x56x256	800k	589824
C3-256	56x56x256	800k	589824
Pool2	28x28x256	200k	0

Layer	Size	Memory	Params
C3-512	28x28x512	400k	1179648
C3-512	28x28x512	400k	2359296
C3-512	28x28x512	400k	2359296
Pool2	14x14x512	100k	0
C3-512	14x14x512	100k	2359296
C3-512	14x14x512	100k	2359296
C3-512	14x14x512	100k	2359296
Pool2	7x7x512	25k	0
FC	1x1x4096	4096	102760448
FC	1x1x4096	4096	16777216
FC	1x1x1000	1000	4096000

- Total memory: 24M * 4 bytes $\sim=$ 96MB
- Total params: 138M

GoogleNet

- Winner for for 2014, 6.7% error for top-5
- 22 Layers
- Only 5 million parameters
- 12X less than AlexNet, 27X less than VGG16
- Efficient inception module
- No FC layers

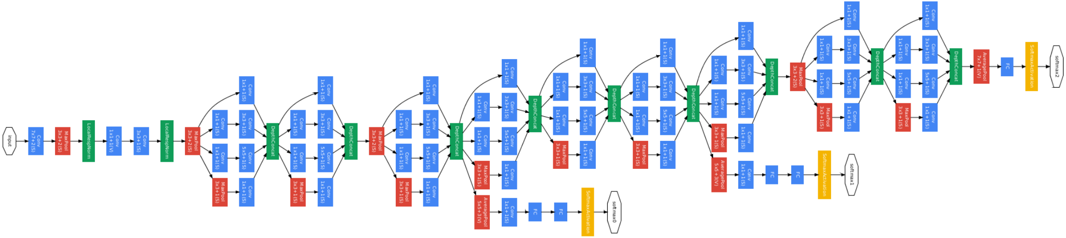
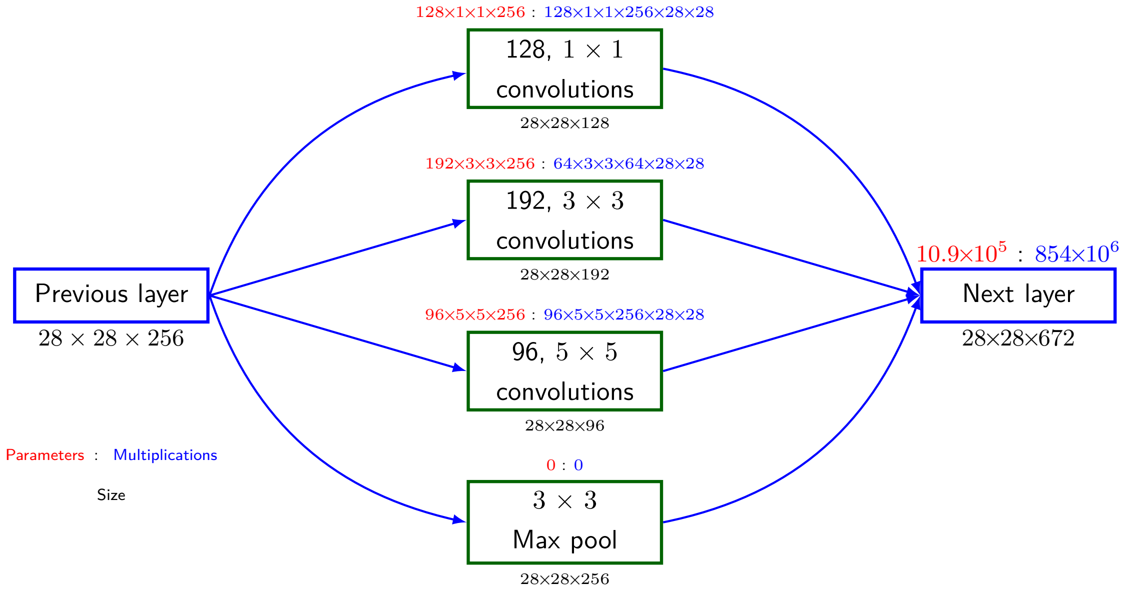
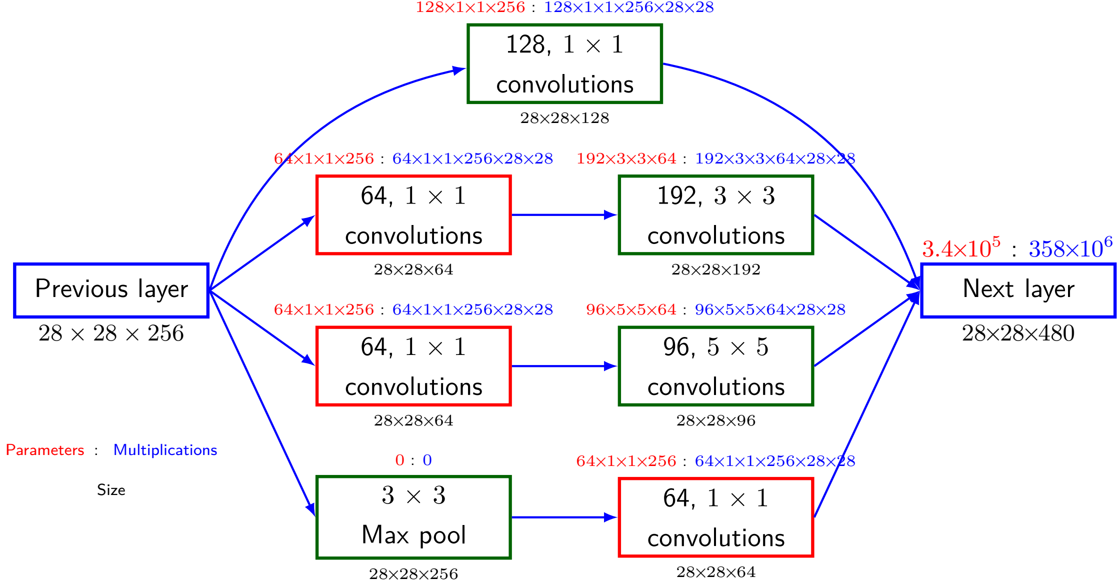


Image source: http://joelouismarino.github.io

Naive inception



Inception


128×1×1×256 : 128×1×1×256×28×28
128, 1 × 1 convolutions
28×28×128
64×1×1×256 : 64×1×1×256×28×28
64, 1 × 1 convolutions
28×28×64
192×3×3×64 : 192×3×3×64×28×28
192, 3 × 3 convolutions
28×28×192
Previous layer
28 × 28 × 256
64×1×1×256 : 64×1×1×256×28×28
64, 1 × 1 convolutions
28×28×64
96×5×5×64 : 96×5×5×64×28×28
96, 5 × 5 convolutions
28×28×96
3.4×10^5 : 358×10^6
Next layer
28×28×480
Parameters : Multiplications
Size
0 : 0
3 × 3 Max pool
28×28×256
64×1×1×256 : 64×1×1×256×28×28
64, 1 × 1 convolutions
28×28×64

ResNet: Observation

- Winner for for 2015, 3.57% error for top-5
- 152 Layers

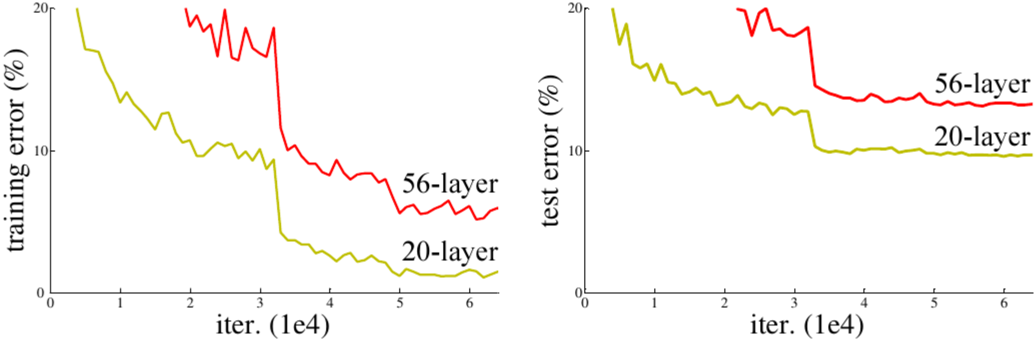


Image source: internet

ResNet

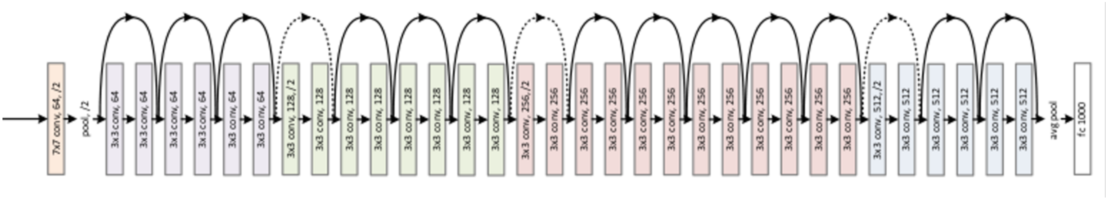
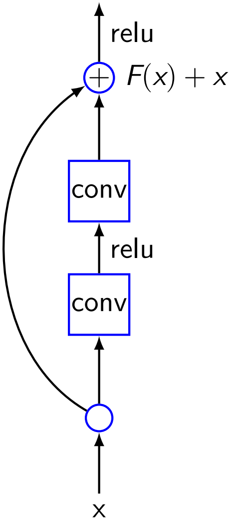
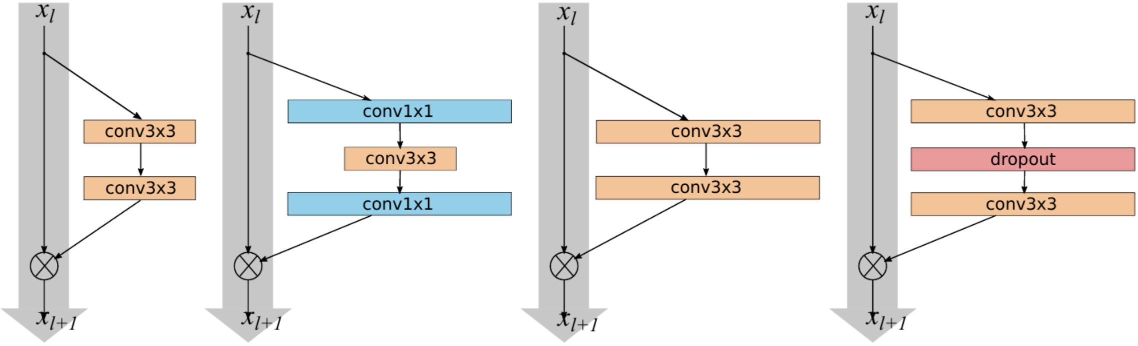


Image source: internet

Resnet: Skip Connection



Wide Resnet



(a) basic (b) bottleneck (c) basic-wide (d) wide-dropout

Image source: Wide Residual Networks

DenseNet

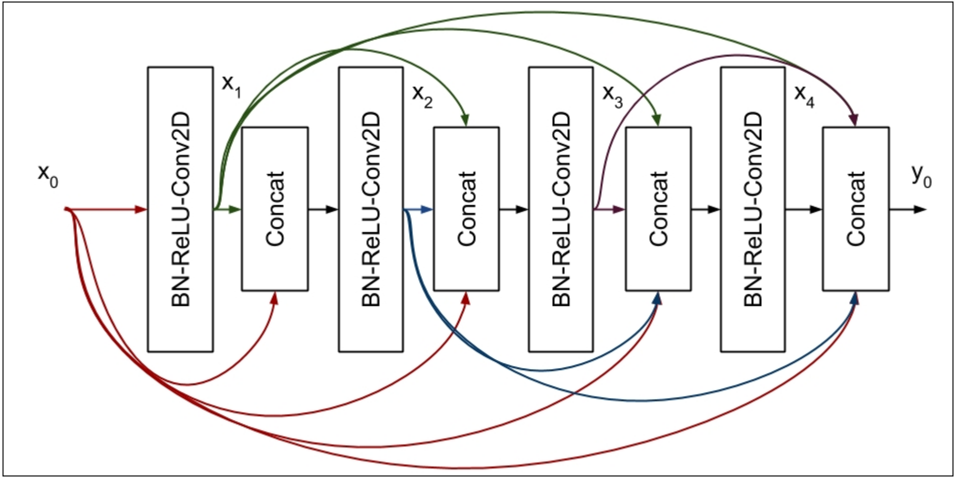


Image source: internet

Comparison of CNN architecture

Model	Size (M)	Top-1/top-5 error (%)	# layers	Model description
AlexNet	238	41.00/18.00	8	5 conv + 3 fc layers
VGG-16	540	28.07/9.33	16	13 conv + 3 fc layers
VGG-19	560	27.30/9.00	19	16 conv + 3 fc layers
GoogleNet	40	29.81/10.04	22	21 conv + 1 fc layers
ResNet-50	100	22.85/6.71	50	49 conv + 1 fc layers
ResNet-152	235	21.43/3.57	152	151 conv + 1 fc layers

Image source: internet

Computer Vision Tasks

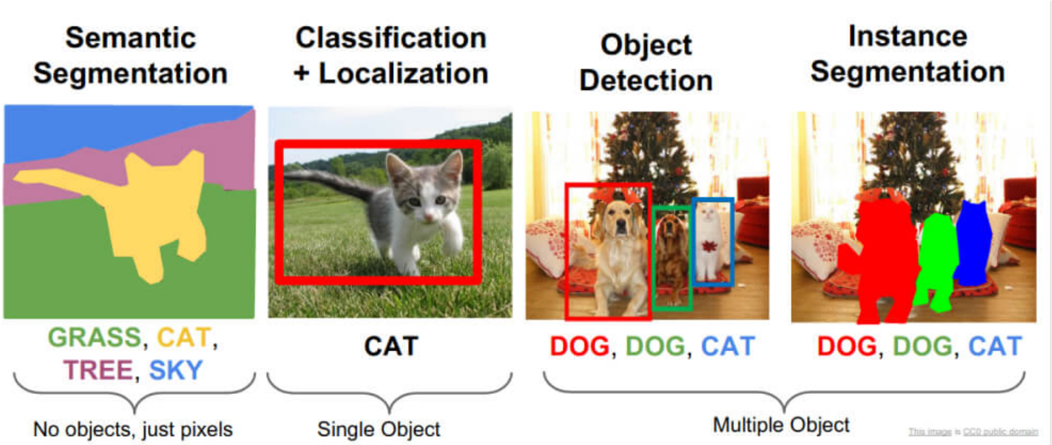
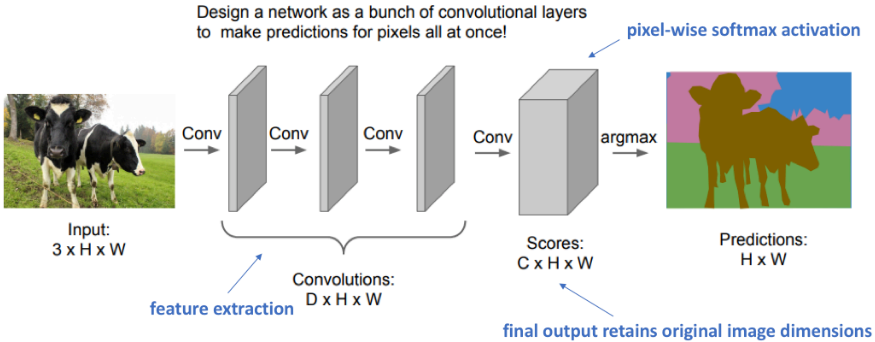


Image source: internet

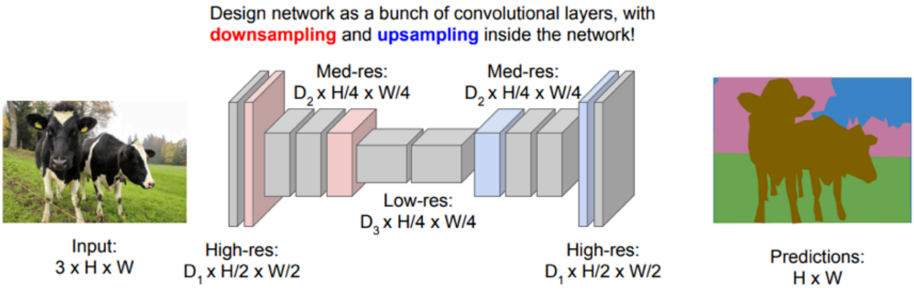
Semantic Segmentation-1



Downside: Preserving image dimensions throughout entire network will be computationally expensive.

Image source: internet

Semantic Segmentation-2



Solution: Make network deep and *work at a lower spatial resolution* for many of the layers.

Image source: internet

Semantic Segmentation-3

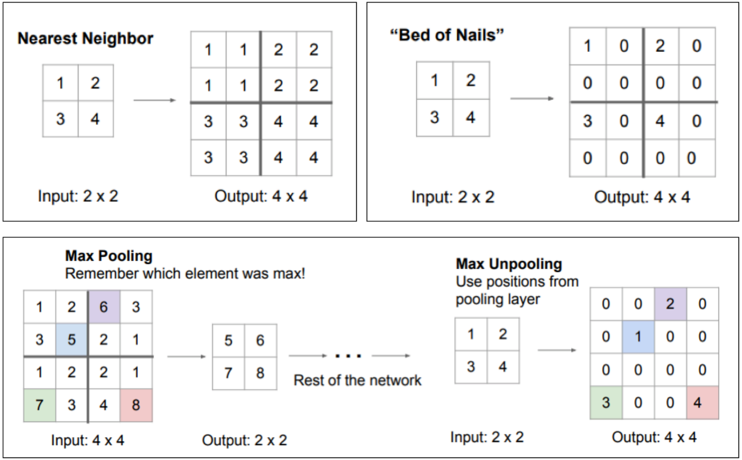


Image source: internet

Object identification: R-CNN

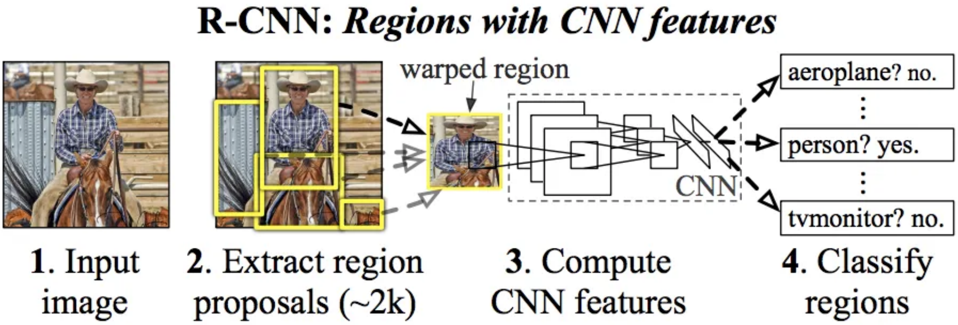


Image source: internet

Object identification: R-CNN

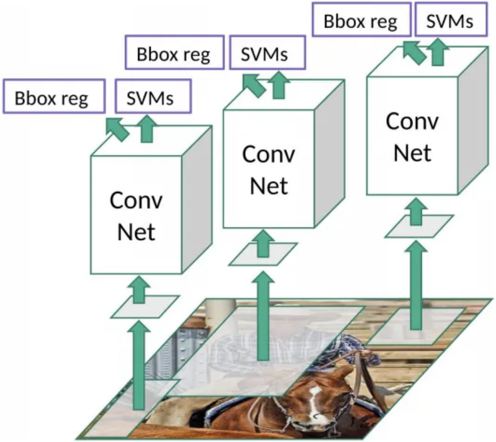


Image source: internet

Object identification: Fast R-CNN

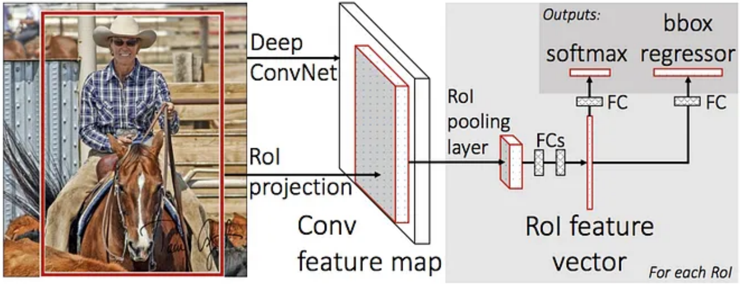


Image source: internet

Object identification: Faster R-CNN

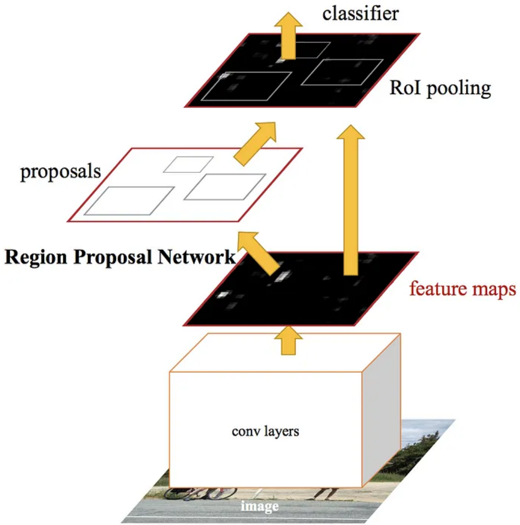
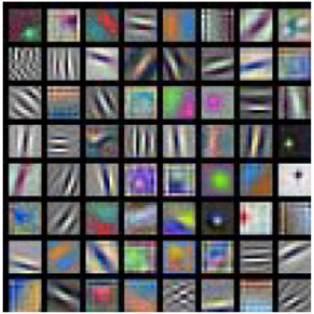


Image source: internet

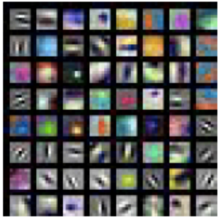
CNN: Visualization

- Visualization of fliters
- Visualization of last layer features
- Visualization of activations
- Identifying important pixels
- Saliency map
- Image synthesis
- Style transfer

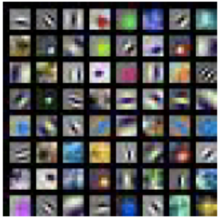
CNN: Visualization of Filters



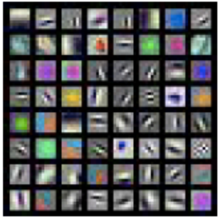
AlexNet:
64 x 3 x 11 x 11



ResNet-18:
64 x 3 x 7 x 7



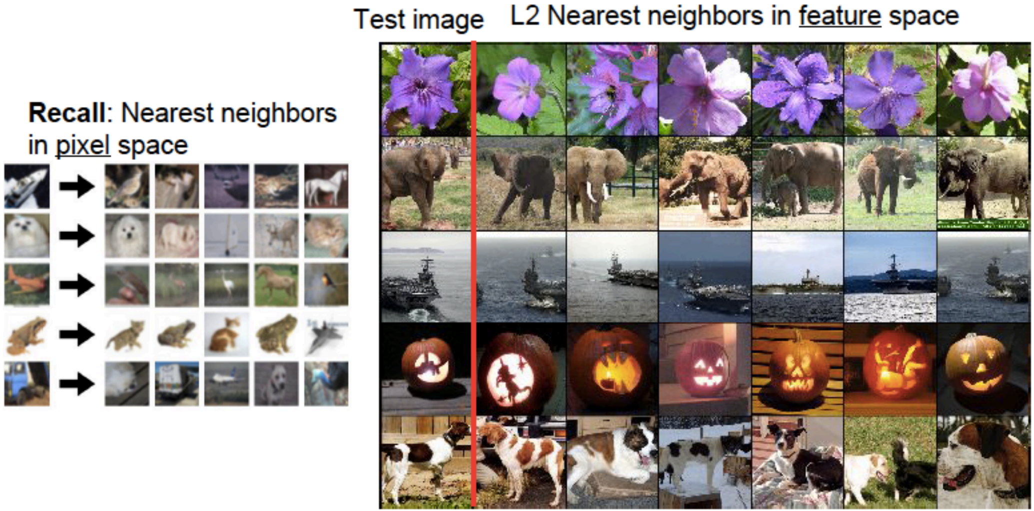
ResNet-101:
64 x 3 x 7 x 7



DenseNet-121:
64 x 3 x 7 x 7

Image source: https://srdas.github.io/DLBook/ConvNets.html

CNN: Nearest neighbour



Krizhevsky et al, "ImageNet Classification with Deep Convolutional Neural Networks", NIPS 2012.
Figures reproduced with permission.

Image source: https://srdas.github.io/DLBook/ConvNets.html

t-SNE: Last layer feature

- t-distributed stochastic neighbor embedding

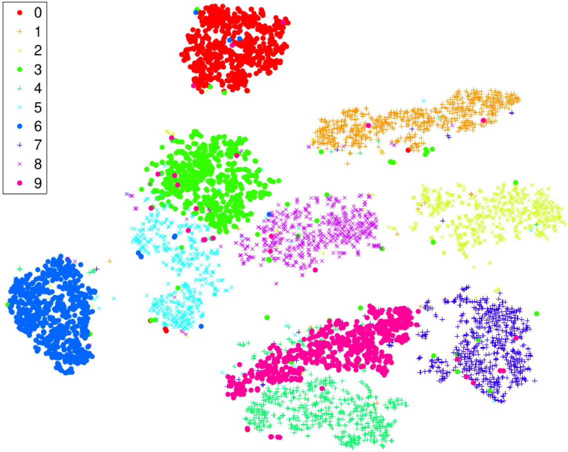
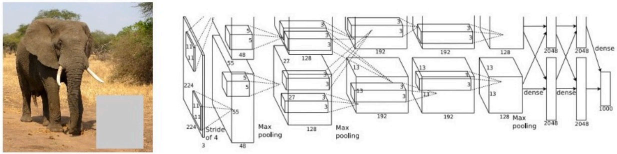


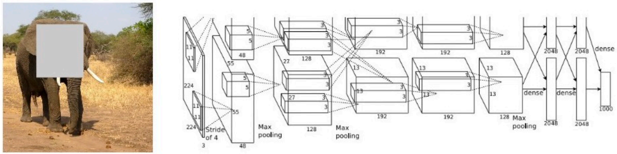
Image source: https://srdas.github.io/DLBook/ConvNets.html

Saliency via Occlusion

Mask part of the image before feeding to CNN, check how much predicted probabilities change



P(elephant) = 0.95



P(elephant) = 0.75

Zeiler and Fergus, “Visualizing and Understanding Convolutional Networks”, ECCV 2014

Image source: https://aman.ai/cs231n/visualization/

CNN: Saliency via Backprop

- Find $\frac{\partial o}{\partial x_i}$. Eg. for imagenet, we will have 224x224x3
- Take absolute value and max over RGB channel
- Image will reduced to 224x224x1

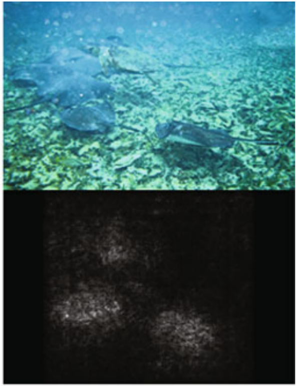


Image source: Neural Networks and Deep Learning

CNN: Guided Backpropagation

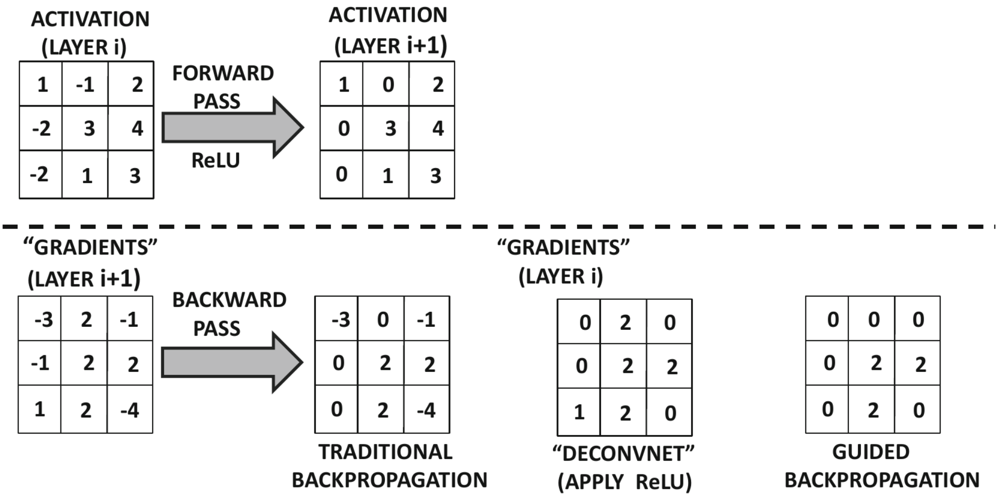
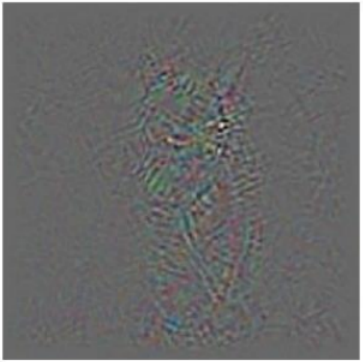


Image source: Neural Networks and Deep Learning

Guided backpropagation



Backprop

Guided Backprop

Image source: internet

Guided backpropagation

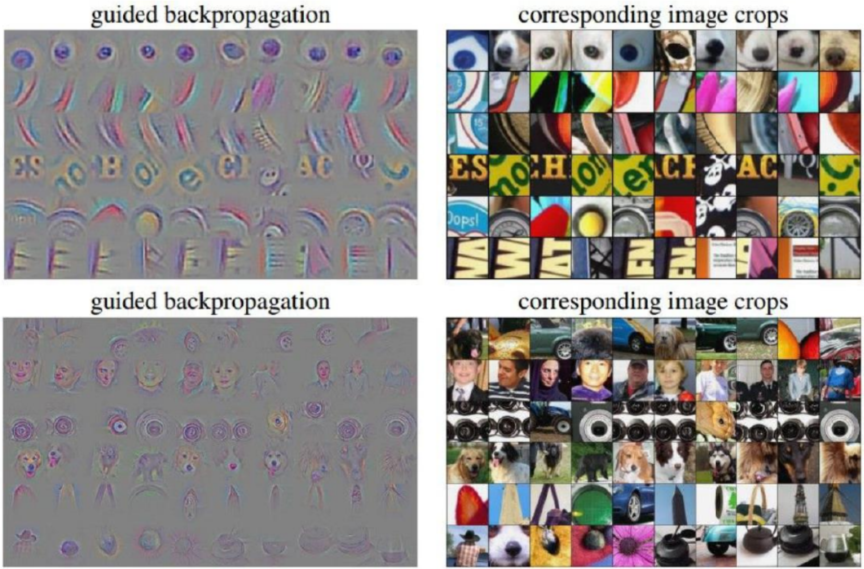


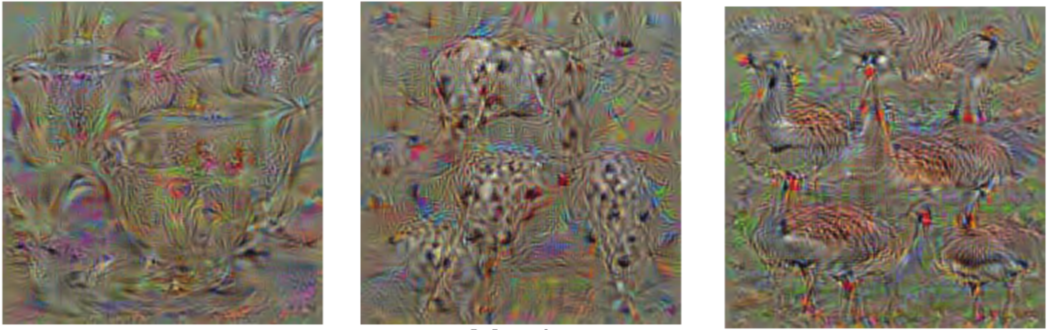
Image source: Neural Networks and Deep Learning

Gradient Ascent

- What kind of image maximizes an activation in a neuron?
- What kind of an image achieves the maximum score for a specific category/class?
- With gradient ascent, we are trying to learn the image that maximizes the activations for a particular class: $I^* = \arg\max_I f(I) + R(I)$, f - neuron value, R - regularizer
- In summary, here are the steps of gradient ascent:
  - Initialize image to zeros.
  - Forward image to compute current scores.
  - Backprop to get gradient of the neuron activation with respect to the input image pixels.
  - Make a small update to the image.

Fantasy image

- Find an input that maximizes a particular neuron at the output (before softmax)



cup dalmatian goose

Image source: Neural Networks and Deep Learning

Adversarial Perturbations

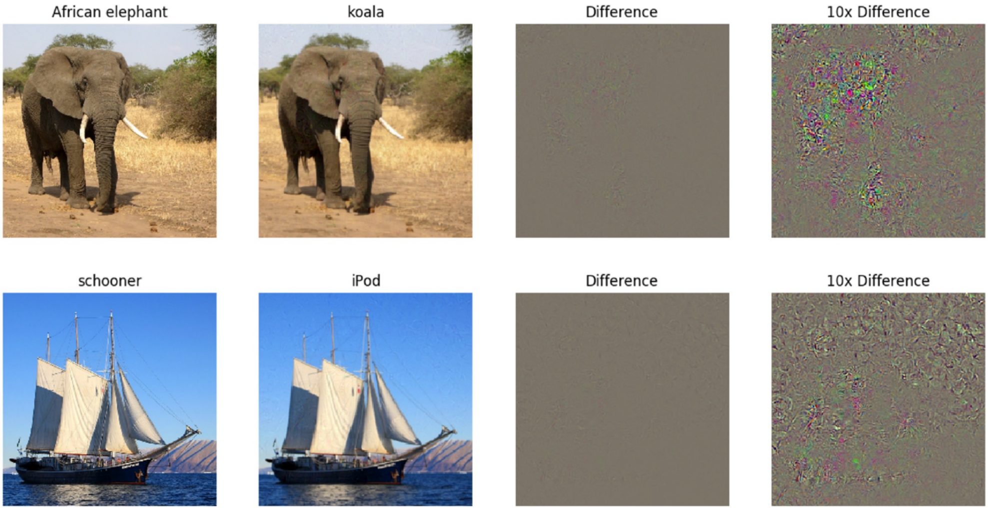


Image source: internet

Universal Adversarial Perturbations

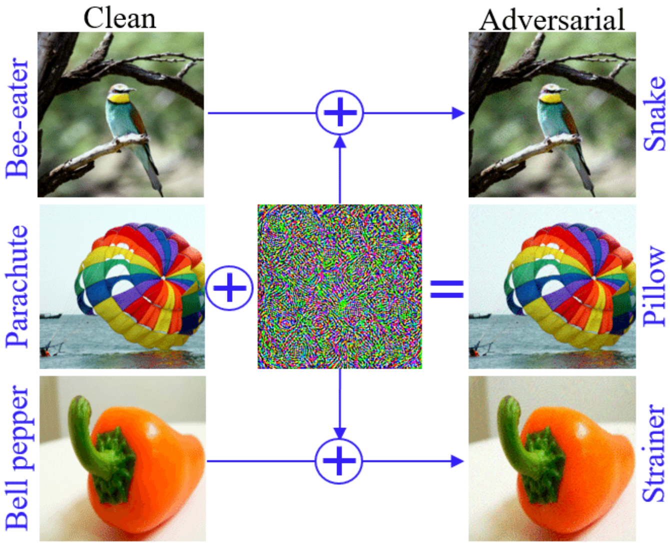


Image source: internet

Deep Dream-1

- Start with a trained CNN. Weight values are fixed. Select some image from the test set.
- Do a forward pass through the network and compute the activations at all the nodes, up until a chosen layer.
- Set the gradients at each node of the chosen layer equal to the activation at that node, i.e., $\frac{\partial L}{\partial h_i} = h_i$
- Using the Backprop algorithm compute the gradients $\frac{\partial L}{\partial x_{ijk}}$
- Change the pixel value to $x_{ijk} = x_{ijk} - \eta \frac{\partial L}{\partial x_{ijk}}$
- Add the training set mean image to the final pixel values to obtain the final image.

Deep Dream-2

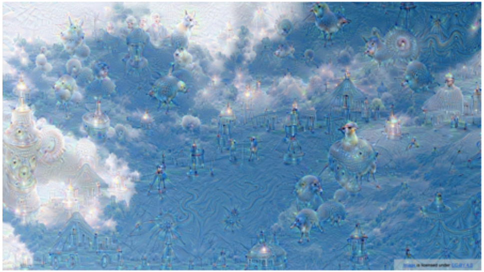


Image source: internet

Deep Dream-3

Image source: internet

Feature Inversion

- Generation of image whose feature vector is specified
- $\mathcal{L} = \|\phi(X) - \phi_0\| + \lambda R(X)$, ϕ_0 is the feature vector

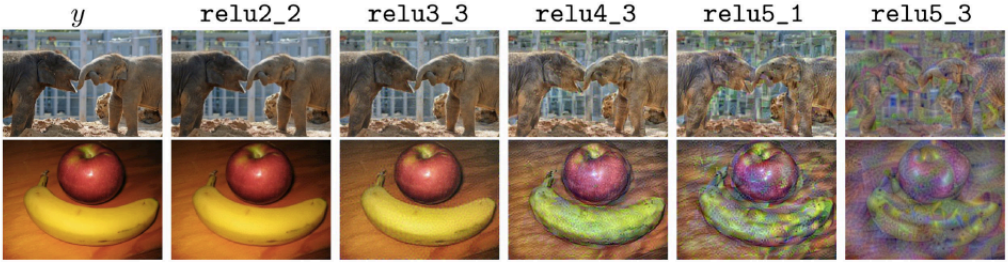


Image source: internet

Resources

- URL: https://cs.stanford.edu/people/karpathy/convnetjs/
- URL: https://playground.tensorflow.org