A Deductive Reasoning Approach for Database Applications using Verification Conditions

Md Imran Alam[a], Raju Halder[a] and Jorge Sousa Pinto[b]

[a]Indian Institute of Technology Patna, India
[b]HASLab/INESC TEC, Universidade do Minho, Braga, Portugal

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ABSTRACT

Deductive verification has gained paramount attention from both academia and industry. Although intensive research in this direction covers almost all mainstream languages, the research community has paid little attention to the verification of database applications. This paper proposes a comprehensive set of Verification Conditions (VCs) generation techniques from database programs, adapting Symbolic Execution, Conditional Normal Form, and Weakest Precondition. The validity checking of the generated VCs for a database program determines its correctness w.r.t. the annotated database properties. The developed prototype `DBverify` based on our theoretical foundation allows us to instantiate VC generation from PL/SQL codes, yielding to detailed performance analysis of the three approaches under different circumstances. With respect to the literature, the proposed approach shows its competence to support crucial SQL features (aggregate functions, nested queries, NULL values, and set operations) and the embedding of SQL codes within host imperative language. For the chosen set of benchmark PL/SQL codes annotated with relevant properties of interest, our experiment shows that only 38% procedures are correct, while 62% procedures violate either all or part of the annotated properties. The primary cause for the latter case is mostly due to the acceptance of runtime inputs in SQL statements without proper checking.

1. Introduction

Static program verification is an integral part of the software engineering process to formally prove or disprove the correctness of programs without executing them. Acknowledging its immense importance in critical systems, intensive research in this direction has been taking place since its inception 45 years ago [1] and a rich class of verification methods, such as Theorem Proving [2, 3, 4], Model Checking [5, 6, 7], Process Algebra [8], have been successfully introduced to formally verify the correctness of both finite-state (e.g., hardware designs) and infinite-state (e.g., software programs) systems, guaranteeing that an implementation or design satisfies its specification.

Deductive reasoning, based on general-purpose theorem proving, has emerged as a promising technique for software verification. Intensive research in this direction has been taking place with a coverage of almost all mainstream languages [2, 4, 9, 10, 11]. Presently, the field of deductive verification has reached a stage of maturity for its use in an industrial setting [12]. In general, the deductive approach internally employs a Verification Condition Generators (VCG) which takes, as input, a program with specification and output a number of proof obligations, called Verification Conditions (VCs). The VCs are then sent to a backend proof tool for validity checking [13]. The use of VCs provides a relatively complete proof system with the use of a richer first-order specification language.

Database applications play a pivotal role in every aspect of our daily lives. Their existence is realized everywhere, ranging from simple web applications to even critical systems like banking, e-commerce, e-government, health-care, etc. In many situations, organizations prefer to adopt third party software modules and integrate them into their existing database-driven systems, keeping the underlying databases intact. Therefore, verification of such untrusted modules is essential, as erroneous codes may lead to inconsistency in the existing database data by violating their properties of interest. Figure 1 exemplifies such a scenario considering a budget allocation system: The attributes TA of the database table BudgetTab stores total proposed budget for each department, whereas other attributes MP, EQ, CT, and CS maintain its distribution under four heads *Manpower*, *Equipment*, *Contingency*, and *Consumable* respectively. The procedure DBprog extracts TA into the application variable z for a given department y (at program point 6) and compares it with the department's available budget x (at program point 7). The statement at 10 adjusts the budget by subtracting an equal fraction of the deficit amount (z - x) among four heads. The specification containing the number of constraints or properties in the form of `CHECK` constraints is specified as part of the BudgetTab table definition, which must be respected by the procedure DBprog on all executions. However, observe that as DBprog accepts run time inputs, the update operation may lead to a violation of the specification by reducing budget amount under some heads below their minimum threshold specified in the `CHECK` constraints.

1.1. Motivation and Contributions

The presence of external database states, along with programs internal states, makes the verification task of database applications more challenging and painstaking one. We ob-

E-mail addresses: imran.pcs16@iitp.ac.in (M. I. Alam), halder@iitp.ac.in (R. Halder), jsp@di.uminho.pt (J. S. Pinto) (M.I. Alam)
ORCID(s):

```
CREATE TABLE BudgetTab
(
    Did INT PRIMARY KEY,
    Dname VARCHAR(50),
    TA NUMBER NOT NULL,
    MP NUMBER NOT NULL,
    EQ NUMBER NOT NULL,
    CT NUMBER NOT NULL,
    CS NUMBER NOT NULL,
    // Set of properties
    CHECK (MP >= 80000),
    CHECK (EQ >= 60000),
    CHECK (CT >= 10000),
    CHECK (CS >= 5000)
);
```

(a) Table definition

```
1. CREATE OR REPLACE PROCEDURE DBprog (y int, x int) IS
2. z int;
3. m int;
4. n int;
5. BEGIN
6.    SELECT TA INTO z FROM BudgetTab
      WHERE Did = y;
7.    if (z ⩾ x) then
8.        m := z − x;
9.        n := m/4;
10.       UPDATE BudgetTab SET MP = MP − n,
          EQ = EQ − n, CT = CT − n,
          CS = CS − n WHERE Did = y;
11.   endif;
12. end;
```

(b) Stored procedure DBprog

Figure 1: Motivating Example

served that, although the database is an integral and indispensable part of most computing environments today, the research community has paid little attention in this direction.

On searching exhaustively in the literature, we find relatively few numbers of attempts to verify database applications [14, 15, 16, 17, 18]. Let us briefly present them. The proposed work in [17] manually translates the embedded SQL code into SmpSL script language and then computes verification conditions using the weakest precondition. Unfortunately, the approach suffers from a severe limitation in terms of scalability to adopt it for real-world programs. Precisely, the verification process takes care of only the decidable fragment of the problem within the scope of the two-variable first-order logic formula. As a result, the approach fails to accept SQL codes in the presence of arithmetic operations, aggregate functions, JOIN operations, etc. Predicate abstraction-based integrity constraints verification of extended version of O_2 object-oriented database language is proposed in [15]. In order to cope with the verification complexity due to object referencing, the source code is translated into an intermediate form on which the predicate abstraction is applied. We observed that the intermediate language is not expressive enough to accommodate important database language features, such as nested query, arithmetic expressions, aggregate functions, etc. Authors in [16] propose integrity constraints verification for database applications using transformation operators. The proposed approach expressed every update operation as a predicate $\mathbf{U} = P(\vec{a})$ and integrity constraints defined in a constraint theory τ. The function $\mathbf{after}^U(\tau)$ translates the constraint theory to the weakest precondition of ϕ with respect to the update $\mathbf{U}$, and a simplified formula is obtained by applying function $\mathbf{Simp}^U(\phi)$. Verification of RDBMS specification and implementation is proposed in [18] using Coq proof assistant, which has expressive-power limited to relational algebra only. The proposal in [14] allows transactions to write in a functional language F# with database table-definitions as refinement types, and verify them using the refinement-type checker Stateful F7. In contrast to [18], where the main concern is to address database implementation issues, the approach in [14] mainly concerns bugs in the user-defined transactions. A comparative summary w.r.t. the literature is depicted in Table 1, where the existing proposals are compared w.r.t. ours based on their potential to deal with the type of properties, NULL values, aggregate functions, arithmetic expressions, and language paradigms. The notations T1-T5 represent the type of properties [19] as follows: T1: Attribute-based (properties involving single attribute), T2: Tuple-based (properties involving multiple attributes), T3: Properties involving NULL values, T4: Properties involving aggregate values, and T5: General Assertions (properties involving both attributes and application variables).

Table 1
Comparative summary w.r.t. the literature

Proposals	Properties	NULL	Aggregate Functions	Arithmetic Expressions	Language
Itzhaky et. al [17]	T1, T2	No	No	No	SQL + Imperative
Baltopoulos et. al [14]	T1-T3	Yes	No	Yes	SQL
Malecha et. al [18]	T2	No	Yes	No	SQL
Benzaken et. al [15]	T2	No	No	No	O_2
Christiansen et. al [16]	T2	No	No	No	SQL
Our proposal	T1-T5	Yes	Yes	Yes	SQL + Imperative

This should be noticed that the applicability of the existing solutions in the literature is relatively poor due to their inability to embrace most crucial SQL features such as aggregate functions, nested queries, NULL values, arithmetic expressions. Moreover, most of them focus only on the verification of SQL statements without the support of the imperative language paradigm. In order to compensate the extra complexity, the approaches follow a common step of translating SQL into an intermediate form, and therefore the expressiveness power of the intermediate language often imposes a limitation on the verification of SQL features. Usually, database applications are written in popular host programming languages such as C, C++, Java, etc., with embedded data access logic expressed declaratively in Structured Query Language (SQL) or SQL-derived programming languages such as PL/SQL and T/SQL [20]. Therefore, verification of such applications, in contrast to the programs in mainstream languages, demands a different treatment due to the presence of database attributes along with program variables.

In order to facilitate the correctness proof of database applications addressing the above-mentioned challenges, in

this paper, we propose a comprehensive set of techniques for the generation of Verification Conditions (VCs) from database programs, adapting Symbolic Execution (SE), Conditional Normal Form (CNF) and Weakest Precondition (WPC). If the generated VCs for a given database program can be discharged (i.e., proved valid by automated theorem prover), then the program is guaranteed to be correct w.r.t. the specified database properties. The developed prototype `DBverify` based on our theoretical foundation allows us to instantiate VC generation from a set of PL/SQL benchmark codes [21, 22, 23, 24, 25, 26, 27, 28], yielding to detailed performance analysis of these three approaches under different circumstances. As reported in Table 1, with respect to the literature, our approach is powerful enough to verify SQL codes embedded within a host imperative language, with a coverage of the above-mentioned crucial SQL features and common database properties [19]. This is worth mentioning that our work is primarily motivated by [13, 29, 30].

To summarize, the main contributions in this paper are:

- We propose a comprehensive set of techniques to generate Verification Conditions (VCs) from database applications where database statements are embedded into a host imperative language. The generated VCs are then processed by an automated theorem prover for their validity checking in order to prove the program's correctness. To this aim, we adapt Symbolic Execution (SE), Conditional Normal Form (CNF), and Weakest Precondition (WPC). The proposed techniques allow to support important SQL features, including aggregate functions, nested queries, NULL values, and various operations (JOIN, UNION, INTERSECT, and MINUS).
- We formalize the conversion of database programs into a single assignment form, which facilitates the verification process, especially in the case of Symbolic Execution and Conditional Normal Form.
- We develop DBverify, a verification tool implemented in Python, based on our theoretical foundation, which enables users to verify PL/SQL procedures under three different approaches. DBverify makes use of ANTLR parser [31] to generate VCs and Microsoft's Z3 theorem prover [32] to check the validity.
- Finally, we perform an experimental evaluation on a set of benchmark PL/SQL codes [21, 22, 23, 24, 25, 26, 27, 28] under all three different approaches. We present a detailed performance analysis based on the experimental results and establish the effectiveness of the approaches under various circumstances.

The rest of the paper is organized as follows: Section 2 recalls the abstract syntax of the languages under consideration and introduces the conversion of database programs into single assignment form. Section 3 presents in detail all the proposed verification condition generation techniques for database programs. The complexity analysis of the proposed approaches and their correctness proofs are detailed in Section 4. Section 5 presents DBverify, a prototype implementation for the verification of PL/SQL codes. The experimental result on a set of PL/SQL benchmark codes with detailed performance analysis is described in Section 6. Section 7 presents threats to validity. Finally, Section 8 covers the current state-of-the-art in this research line, and Section 9 concludes our work.

2. Database Language and Single Assignment Form

In this section, we first recall from [33, 34] the abstract syntax of database language under consideration. Then we introduce its single assignment form, an intermediate language representation, which serves as a backbone of some approaches proposed in Section 3.

2.1. Abstract Syntax of Database Language

We consider a generic scenario of database applications where SQL codes are embedded into another high-level host language. To this aim, let us recall from [33, 34] the abstract syntax of the language, which, for the sake of simplicity in the theoretical formalism, supports SQL data manipulation languages hosted by imperative statements. This is depicted in Table 2. The variables are categorized into two: database attributes set $\mathbb{V}_d$ and application variables set $\mathbb{V}_a$. The arithmetic expressions e and boolean expressions b are defined accordingly, considering the presence of either $v \in \mathbb{V}_a$ or $a \in \mathbb{V}_d$ or both along with possible arithmetic and/or relational operators. Observe that g, r, f, and s represent group-by, distinct/all, order-by, and aggregate functions respectively, where id is the identity function.

The SQL statements Q_{sel}, Q_{upd}, Q_{ins}, and Q_{del} consist of an action-part and a condition-part. For example, in the update statement $\langle \vec{a} := \texttt{UPDATE}(\vec{e}),\ cond \rangle$, the first component $\vec{a} := \texttt{UPDATE}(\vec{e})$ represents an action-part and the second component $cond$ represents a condition-part which follows well-formed formulas in the first order logic. To exemplify this, let us consider the statement "`UPDATE` emp `SET` sal := sal + *bonus* `WHERE` age $\geqslant$ 60", where "age $\geqslant$ 60" represents condition part and $\langle \text{sal} \rangle := \texttt{UPDATE}(\langle \text{sal} + bonus \rangle)$ represents action part. Therefore, its abstract syntax is defined as $\langle \langle \text{sal} \rangle := \texttt{UPDATE}(\langle \text{sal} + bonus \rangle), \text{age} \geqslant 60 \rangle$. Informally, the semantics of the update statement can be described as follows: For the database tuples which satisfy the condition-part $cond$, the values of the attributes $a_i \in \vec{a}$ are updated (in sequence) by $e_i \in \vec{e}$. Similarly, given a query "`SELECT DISTINCT` Dno, Pno, `MAX`(Sal) `INTO` *rs* `FROM` Tab `WHERE` Sal > 1000 `GROUP BY` Dno, Pno `HAVING MAX`(Sal) < 4000 `ORDER BY` Dno". Its abstract syntax, according to Table 2, is $\langle rs := \texttt{SELECT}(f(\vec{e}),\ r(\vec{h}(\vec{a})),\ \phi,\ g(\vec{e}),\ cond) \rangle$, where $cond \triangleq \text{Sal} > 1000$, $g(\vec{e}) \triangleq \texttt{GROUP BY}(\langle \text{Dno, Pno} \rangle)$, $r(\vec{h}(\vec{a})) \triangleq \texttt{DISTINCT}(\langle \texttt{DISTINCT}(\text{Dno}),\ \texttt{DISTINCT}(\text{Pno}),\ \texttt{MAX} \circ \texttt{ALL}(\text{Sal}) \rangle)$, $\phi \triangleq \texttt{MAX}(\text{Sal}) < 4000$, and $f(\vec{e}) \triangleq$ `ORDER`

BY ASC(⟨Dno⟩). In general, ϕ filters a set of tuples from the target table based on the satisfiability of *cond* and the result obtained after processing these tuples using g, ϕ, h, r, f (if present) is stored in the resultset application variable *rs*. The syntax of *cond* in the form of $a \otimes (Q_{sel})$ supports nested query as well. Observe that, since insertion of a new tuple does not require any condition to satisfy, the *cond* in Q_{ins} is by default $false$.

2.2. Assertion Language and Database Properties

As deductive verification is not fully automatic, this requires annotating the source code by *assume* and *assert* according to the specifications. The informal semantics of *assume* and *assert* are as follows: *assume* ψ command excludes all computations which do not satisfy logical expression ψ, whereas *assert* ψ commands at different program points specify the properties to be proven. A program is correct if, for every execution, whenever *assert* ψ is reached, the assertion ψ is satisfied by the current state. Since our verification starts with the assumption that the initial database is in consistent states w.r.t database properties, in order to reflect this, we annotate our program by placing *assume* command at the beginning of the code. As the database commands are responsible for change database states, to detect any property violation at more granularity level, we allow the annotation by placing *assert* commands anywhere within the program, especially after database commands.

We adopt the assertion language from [35], defined in Equation 1, in order to support the following types of database properties [19]: Attribute-based, Tuple-based, Properties involving NULL and aggregate values, and General Assertions.

$$\psi ::= true \mid false \mid a_0 = a_1 \mid a_0 \leqslant a_1 \mid \psi_0 \vee \psi_1 \mid \psi_0 \wedge \psi_1 \mid \neg\psi \mid \psi_0 \Rightarrow \psi_1 \mid \forall i.\psi \mid \exists i.\psi \quad (1)$$

where i ranges over integer variables and a is the arithmetic expression defined below:

$$a ::= n \mid i \mid a_0 + a_1 \mid a_0 - a_1 \mid a_0 \times a_1$$

where $n \in \mathbb{R}$. Readers may refer to [35] for the semantics of the assertion language.

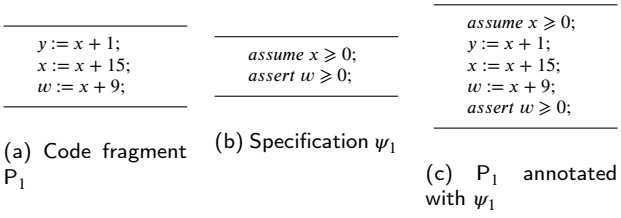
To exemplify various property types, let us consider a database schema: `Employee(ID, Name, DOJ, Experience, Salary)`, `Department (DID, Dname, MgrID, MgrStartDate)`. The following assertions '`Salary` $> 100 \wedge$ `Salary` $\leqslant 4000$' and '`Experience` $> 5 \wedge$ `Salary` > 2000' represent attribute-based and tuple-based properties respectively depending upon the presence of one or more attributes in the assertions, whereas the assertion '`MgrStartDate` $>$ `DOJ` $\wedge$ `Experience` $>$ y' represents a general assertion due to the involvement of both database attributes and application variables. The examples of assertions involving NULL and aggregate values are '`DID NUMBER NOT NULL`', 'Avg(Salary) > 2500' respectively.

2.3. Single Assignment Form of Database Language

Single Assignment [36] is a semantically equivalent representation of a program in which every new assignment to variable results into a new version and each version denotes a different logical variable. This ensures that, in single assignment, each variable is defined only once before being used. A versioned variable is defined by the original name of the variable associated with an integer subscript representing its current version.

As logical encoding (known as verification condition (VC)) of database programs is a prime objective of deductive-based verification. Let us demonstrate on a simple code snippet how a single assignment form of program code leads to the generation of the correct form of VCs capturing the actual program semantics. Given the code fragment P_1 and the specification ψ_1 depicted in Figures 2(a) and 2(b). The logical expression $f \triangleq x \geqslant 0 \wedge y = x + 1 \wedge x = x + 15 \wedge w = x + 9 \implies w \geq 0$ represents a VC of the annotated code in Figure 2(c).

However, observe that, although P_1 is correct w.r.t. ψ_1, this can never be captured due to the unsatisfiability of f because $x = x + 15$ is always unsatisfiable. To make f satisfiable, one has to differentiate between the variable x appears at the left and right sides of the assignment operator (:=). The best way to do this is to convert program codes into a single assignment form [13].



(a) Code fragment P_1 (b) Specification ψ_1 (c) P_1 annotated with ψ_1

Figure 2: A simple code fragment and its annotation

Let us now make a quick journey through a single assignment form of imperative statements, and then we propose the same for our database language under consideration.

Single Assignment Form of Imperative Statements [36, 37]. As the prime objective in single assignment form is to ensure single definitions for all variables, in the case of an assignment statement, a new version for the defined variable appeared on the left side of the assignment operator is introduced. For example, consider the code: {`read` x; $x := x + 1$;}, its equivalent single assignment form is {`read` x_0; $x_1 := x_0 + 1$;}, where x_0 and x_1 denote the initial version and the current version of x respectively. In case of conditional statements where two or more control flow paths merge at a point, the single assignment property may get violated since multiple definitions of a variable may reach at that merging point. To solve this problem, imaginary assignments are introduced at the merging points by using ϕ-functions and the resultant representation is called Static Single Assignment (SSA) form [37]. In general, a ϕ-function has arguments corresponding to each incoming control flow path: the i^{th} argument of a ϕ-function is the incoming value along the i^{th} path. For example, the SSA form of the code fragment P_2 in Figure 3(a) is depicted in Figure 3(b). Note that, in order to make such SSA form executable, the ϕ-function can

Table 2
Abstract Syntax of Database Language [33, 34]

Constant:	c	$\in$	$\mathbb{R}$ (Set of Numerical Constants)
Variables:	v	$\in$	$\mathbb{V}_a$ (Set of Application Variables)
	a	$\in$	$\mathbb{V}_d$ (Set of Database Attributes)
	Var	$::=$	$\mathbb{V}_a \cup \mathbb{V}_d$
	$\vec{a}$	$::=$	$\langle a_1, a_2, \ldots, a_n \rangle$ (Ordered sequence of attributes)
Expressions:	e	$\in$	$\mathbb{E}$ (Set of Arithmetic Expressions)
	e	$::=$	$c \mid v \mid a \mid e \oplus e$, where $\oplus \in \{+, -, *, /\}$
	b	$\in$	$\mathbb{B}$ (Set of Boolean Expressions)
	b	$::=$	$true \mid false \mid e \odot e \mid b \vee b \mid b \wedge b \mid \neg b$ where $\odot \in \{<, \leq, >, \geq, =, \neq\}$
SQL Conditions: (Well-formed first order formula)	τ	$\in$	$\mathbb{T}$ (Set of Terms)
	τ	$::=$	$c \mid a \mid v \mid f_n(\tau_1, \tau_2, ..., \tau_n)$ where f_n is an n-ary function.
	a_f	$\in$	$\mathbb{A}_f$ (Set of Atomic Formulas)
	a_f	$::=$	$\tau_1 == \tau_2 \mid R_n(\tau_1, \tau_2, ..., \tau_n)$ where $R_n(\tau_1, ..., \tau_n) \in \{true, false\}$
	ϕ	$\in$	$\mathbb{W}$ (Set of Well-Formed Formula)
	ϕ	$::=$	$a_f \mid \neg\phi \mid \phi_1 \vee \phi_2 \mid \phi_1 \wedge \phi_2$
	$cond$	$::=$	$true \mid false \mid \phi \mid a \otimes (Q)$, where $\otimes \in \{$`IN, NOT IN, ANY, EXIST, NOT EXIST`$\}$
SQL Functions:	$g(\vec{e})$	$::=$	`GROUP BY`$(\vec{e})$, where $\vec{e} = \langle e_1, ..., e_n \mid e_i \in \mathbb{E} \rangle$
	r	$::=$	`DISTINCT | ALL`
	s	$::=$	`AVG | SUM | MAX | MIN | COUNT` $\mid id$
	$h(e)$	$::=$	$sor(e)$
	$\vec{h}(\vec{x})$	$::=$	$\langle h_1(x_1), ..., h_n(x_n) \rangle$, $\vec{h} = \langle h_1, ..., h_n \rangle$ and $\vec{x} = \langle x_1, ..., x_n \mid x_i = e \vee x_i = * \rangle$
	$f(\vec{e})$	$::=$	`ORDER BY ASC`$(\vec{e})$ `| ORDER BY DESC`$(\vec{e}) \mid id$
Commands:	Q	$\in$	$\mathbb{Q}$ (Set of SQL Statements)
	Q	$::=$	$Q_{sel} \mid Q_{upd} \mid Q_{ins} \mid Q_{del}$
	Q_{sel}	$::=$	$\langle rs :=$ `SELECT`$(f(\vec{e}),\ r(\vec{h}(\vec{x})),\ \phi,\ g(\vec{e}))$, $cond \rangle$
	Q_{upd}	$::=$	$\langle \vec{a} :=$ `UPDATE`$(\vec{e}),\ cond \rangle$
	Q_{ins}	$::=$	$\langle \vec{a} :=$ `INSERT`$(\vec{e}),\ false \rangle$
	Q_{del}	$::=$	$\langle \vec{a} :=$ `DELETE`$(),\ cond \rangle$
	$stmt$	$\in$	$\mathbb{C}$ (Set of Commands)
	$stmt$	$::=$	$Q \mid v := e$ `| skip | if` b `then` $stmt$ `endif`
			`read` v `| if` b `then` $stmt_1$ `else` $stmt_2$ `endif`
			$stmt_1;\ stmt_2$
Programs:	$\mathcal{P}$	$\in$	$\mathbb{P}$ (Set of database programs)
	$\mathcal{P}$	$::=$	$stmt \mid stmt\ ;\ \mathcal{P}$

equivalently be defined in terms of the ternary operator. For example, the statement $x_3 := \phi(x_1, x_2)$; in Figure 3(b) can be defined as $x_3 := x_0 > 0\ ?\ x_1 : x_2$;.

```
read x;
if x > 0 then
    x := x − 1;
else
    x := x + 1;
endif
y = x + 5;
```

(a) Code fragment P_2

```
read x0;
if x0 > 0 then
    x1 := x0 − 1;
else
    x2 := x0 + 1;
endif
x3 := φ(x1, x2);
y1 = x3 + 5;
```

(b) SSA form of P_2

```
read x0;
if x0 > 0 then
    x1 := x0 − 1;
    x3 := x1;
else
    x2 := x0 + 1;
    x3 := x2;
endif
y1 := x3 + 5;
```

(c) DSA form of P_2

Figure 3: `If-else` statement and its single assignment form

Dynamic Single Assignment (DSA) form is an alternative representation where variables are defined along all control paths to ensure that a single definition reaches towards the merging points. Since the meaning of a ϕ-function is a mapping of all incoming values to a single name (say x_3), it is equivalent to place a copy of x_3 at the end of each predecessor-block. The copy moves the values corresponding to the appropriate ϕ-function argument into x_3. This can be seen as a destruction of a ϕ-function into its direct predecessor-blocks in SSA form [37]. For example, the code snippet in Figure 3(c) depicts the equivalent DSA form of P_2. Similarly, Figure 4 highlights the SSA and DSA treatments of `if`-statement.

Single Assignment Form of SQL Statements. Let us now extend the notions of SSA and DSA to the case of database applications. To exemplify this, consider the `UPDATE` statement Q in Figure 5(a). Treating Q in a similar way as in the case of imperative statements, we get its

```
if (x ⩾ 0) then
    y := y − 5;
endif
x := y;
```

(a) Code fragment P_3

```
if (x0 ⩾ 0) then
    y1 := y0 − 5;
endif
y2 := φ(y1, y0);
x1 := y2;
```

(b) SSA form of P_3

```
if (x0 ⩾ 0) then
    y1 := y0 − 5;
    y2 := y1;
else
    y2 := y0;
endif
x1 := y2;
```

(c) DSA form of P_3

Figure 4: `if`-statement and its single assignement form

single assignment form depicted in Figure 5(b). Note that, in the conversion process, we assume the existence of a ghost database (denoted by cap($\widehat{\ }$) on table names), which contains all versions of the attributes of the original database. This is to observe that the resultant single assignment form would be wrong because it reflects only a part of the database state for which $a_0 > 50 \wedge a_0 < 100$ holds. In order to capture the complete database state, we add an auxiliary `UPDATE` statement by considering the negation of *cond* which covers the other part of the database state. The correct single assignment form of Q is depicted in Figure 5(c). We show this fact pictorially in Figure 6.

The following formula computed from the correct single assignment form of Q represents a valid VC:

$$(b_1 = b_0 - 10 \wedge a_0 > 5 \wedge a_0 < 100) \vee (b_1 = b_0 \wedge \neg(a_0 > 5 \wedge a_0 < 100))$$

Similarly, we can define the single assignment form of `INSERT` and `DELETE` by adding auxiliary `UPDATE` statements before them in order to reflect other parts of the database state also. In these two cases, we have to consider the versioning of all attributes in the table which are referred by the statements. Observe that, the `SELECT` statement changes the

UPDATE T SET $b = b - 10$ WHERE $a > 50$ AND $a < 100$;

(a) UPDATE statement Q

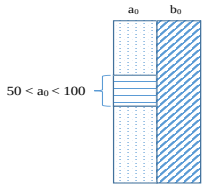
UPDATE $\hat{T}$ SET $b_1 = b_0 - 10$ WHERE $a_0 > 50$ AND $a_0 < 100$;

(b) Wrong single assignment form of Q

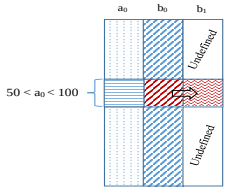
UPDATE $\hat{T}$ SET $b_1 = b_0$ WHERE $\neg(a_0 > 50$ AND $a_0 < 100)$;
UPDATE $\hat{T}$ SET $b_1 = b_0 - 10$ WHERE $a_0 > 50$ AND $a_0 < 100$;

(c) Correct single assignment form of Q

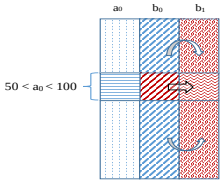
Figure 5: Single assignment form of UPDATE statement



(a) Initial state of table T



(b) State of T w.r.t. single assignment form in Figure 5b



(c) State of T w.r.t. single assignment form in Figure 5c

Figure 6: State representation of the table T w.r.t. Q, and their single assignment form in Figure 5

version of the application result-set variables only.

Treating SQL statements under if- and if-else statements. Like imperative language, the presence of UPDATE or DELETE or INSERT statement in if- and if-else requires special treatment to convert them into SSA or DSA form. The code snippets given in Figure 7 illustrate this.

Observe that the ϕ-function ϕ_d in the case of multi-definitions of database attributes under SSA form can be defined in terms of UPDATE controlled by if or if-else. For example, $b_2 := \phi_d(b_1, b_0)$; at program point 3 in Figure 7(b) can be defined as:

```
if z0 ⩾ 3 then
    UPDATE T̂1 SET b2 = b1;
else
    UPDATE T̂1 SET b2 = b0;
endif
```

```
1.  if z ⩾ 3 then
2.      UPDATE T1 SET b = b − 10 WHERE a = y;
    endif;
3.  SELECT b INTO w FROM T1 WHERE a > 5;
                ⋮
19. if y > 60 then
20.     DELETE FROM T2 WHERE d = y;
    else
21.     INSERT INTO T2 (c, d) VALUES (p, q);
    endif;
22. SELECT d INTO x FROM T2 WHERE c > 5;
```

(a) Original code fragment P_4

```
1.  if z0 ⩾ 3 then
        UPDATE T̂1 SET b1 = b0 WHERE ¬(a0 = y0);
2.      UPDATE T̂1 SET b1 = b0 − 10 WHERE a0=y0;
    endif;
3.  b2 := φd(b1, b0);
4.  SELECT b2 INTO w1 FROM T̂1 WHERE a0 > 5;
                ⋮
19. if y0 > 60 then
        UPDATE T̂2 SET c1=c0, d1=d0 WHERE
        ¬(d0=y0);
20.     DELETE FROM T̂2 (c0, d0) WHERE d0 = y0;
    else
        UPDATE T̂2 SET c2 = c0, d2 = d0;
21.     INSERT INTO T̂2 (c2, d2) VALUES (p0, q0);
    endif;
22. c3 := φd(c1, c2);
23. d3 := φd(d1, d2);
24. SELECT d3 INTO x1 FROM T̂2 WHERE c3 > 5;
```

(b) SSA form of P_4

```
1.  if z0 ⩾ 3 then
        UPDATE T̂1 SET b1 = b0 WHERE ¬(a0 = y0);
2.      UPDATE T̂1 SET b1 = b0 − 10 WHERE a0=y0;
3.      UPDATE T̂1 SET b2 = b1;
    else
4.      UPDATE T̂1 SET b2 = b0;
    endif;
5.  SELECT b2 INTO w1 FROM T̂1 WHERE a0 > 5;
                ⋮
19. if y0 > 60 then
        UPDATE T̂2 SET c1=c0, d1=d0 WHERE
        ¬(d0=y0);
20.     DELETE FROM T̂2 (c0, d0) WHERE d0 = y0;
21.     UPDATE T̂2 SET c3 = c1, d3 = d1;
    else
        UPDATE T̂2 SET c2 = c0, d2 = d0;
22.     INSERT INTO T̂2 (c2, d2) VALUES (p0, q0);
23.     UPDATE T̂2 SET c3 = c2, d3 = d2;
    endif;
24. SELECT d3 INTO x1 FROM T̂2 WHERE c3 > 5;
```

(c) DSA form of P_4

Figure 7: SQL statements and their single assignment form under if- and if-else

On the other hand, in the case of DSA of database code, the else-part is introduced whenever necessary (for example, statement 4 in Figure 7(c)) and ϕ-functions are destructed into their predecessor-blocks (for instance, by introducing statements 3, 4, 21, and 23 in Figure 7(c)), which ensure the single definition of attributes to flow towards the merging points.

Since our proposed verification approaches make use of

$\mathbb{I}$	:	Set of natural numbers	**Versioning Function**	:	$\sigma \in \Sigma \triangleq \mathsf{Var} \rightarrow \mathbb{I}$
$\mathbb{C}$	:	Set of Commands	**Initial Version**	:	$\sigma_0 \triangleq \mathsf{Var} \rightarrow \{0\}$, where $\sigma_0 \in \Sigma$
$\mathbb{C}^{dsa}$	:	Set of Commands in DSA form	**DSA Translation Function**	:	$\mathsf{TranDSA} \triangleq \mathbb{C} \times \Sigma \rightarrow \mathbb{C}^{dsa} \times \Sigma$

$$\mathsf{TranDSA}[\![c]\!]\sigma \triangleq \langle c, \sigma\rangle \qquad \mathsf{TranDSA}[\![v]\!]\sigma \triangleq \langle v_i,\ \sigma\rangle \ when\ \sigma(v) = i \qquad \mathsf{TranDSA}[\![a]\!]\sigma \triangleq \langle a_i,\ \sigma\rangle \ when\ \sigma(a) = i$$

$$\mathsf{TranDSA}[\![e]\!]\sigma \triangleq \langle \forall x \in \mathsf{Var}[\![e]\!] : e\big[\!\big[\mathsf{TranDSA}[\![x]\!]/x\big]\!\big],\ \sigma\rangle \qquad \mathsf{TranDSA}[\![b]\!]\sigma \triangleq \langle \forall x \in \mathsf{Var}[\![b]\!] : b\big[\!\big[\mathsf{TranDSA}[\![x]\!]/x\big]\!\big],\ \sigma\rangle$$

$$\mathsf{TranDSA}[\![\phi]\!]\sigma \triangleq \langle \forall x \in \mathsf{Var}[\![\phi]\!] : \phi\big[\!\big[\mathsf{TranDSA}[\![x]\!]/x\big]\!\big],\ \sigma\rangle \qquad \mathsf{TranDSA}[\![a \otimes (Q_{sel})]\!]\sigma \triangleq \langle \mathsf{TranDSA}[\![a]\!]\sigma \otimes \mathsf{TranDSA}[\![(Q_{sel})]\!]\sigma,\ \sigma\rangle$$

$$\mathsf{TranDSA}[\![assume\ \phi]\!]\sigma \triangleq \langle assume\ \mathsf{TranDSA}[\![\phi]\!]\sigma,\ \sigma\rangle \qquad \mathsf{TranDSA}[\![assert\ \phi]\!]\sigma \triangleq \langle assert\ \mathsf{TranDSA}[\![\phi]\!]\sigma,\ \sigma\rangle$$

$$\mathsf{TranDSA}[\![v\ :=\ e]\!]\sigma \triangleq \langle \mathsf{TranDSA}[\![v]\!]\sigma' := \mathsf{TranDSA}[\![e]\!]\sigma,\ \sigma'\rangle,\ where\ \sigma' = \sigma[\![v \mapsto \sigma(v) + 1]\!]$$

$$\begin{aligned}\mathsf{TranDSA}[\![\langle \vec{a} := \mathtt{UPDATE}(\vec{e}),\ cond\rangle]\!]\sigma &\triangleq \mathsf{TranDSA}[\![\langle \vec{a} := \mathtt{UPDATE}(\vec{a}) \curvearrowright \mathtt{UPDATE}(\vec{e}),\ \neg cond \curvearrowright cond\rangle]\!]\\ &\triangleq \big\langle \langle \mathsf{TranDSA}[\![\vec{a}]\!]\sigma' := \mathtt{UPDATE}(\mathsf{TranDSA}[\![\vec{a}]\!]\sigma) \curvearrowright \mathtt{UPDATE}(\mathsf{TranDSA}[\![\vec{e}]\!]\sigma),\\ &\quad \mathsf{TranDSA}[\![\neg(cond)]\!]\sigma \curvearrowright \mathsf{TranDSA}[\![cond]\!]\sigma\rangle, \sigma'\big\rangle;\ where\ \sigma' = \sigma[\![\vec{a} \mapsto \sigma(\vec{a}) + 1]\!]\end{aligned}$$

$$\begin{aligned}\mathsf{TranDSA}[\![\langle \vec{a} := \langle \mathtt{INSERT}(\vec{e}), false\rangle\rangle]\!]\sigma &\triangleq \mathsf{TranDSA}[\![\vec{a} := \mathtt{UPDATE}(\vec{a}) \curvearrowright \mathtt{INSERT}(\vec{e}),\ true \curvearrowright false\rangle]\!]\\ &\triangleq \big\langle \langle \mathsf{TranDSA}[\![\vec{a}]\!]\sigma' := \mathtt{UPDATE}(\mathsf{TranDSA}[\![\vec{a}]\!]\sigma) \curvearrowright \mathtt{INSERT}(\mathsf{TranDSA}[\![\vec{e}]\!]\sigma),\\ &\quad true \curvearrowright false\rangle, \sigma'\big\rangle;\ where\ \sigma' = \sigma[\![\vec{a} \mapsto \sigma(\vec{a}) + 1]\!]\end{aligned}$$

$$\begin{aligned}\mathsf{TranDSA}[\![\langle \vec{a} := \mathtt{DELETE}(), cond\rangle]\!]\sigma &\triangleq \mathsf{TranDSA}[\![\langle \vec{a} := \langle \mathtt{UPDATE}(\vec{a}_i) \curvearrowright \mathtt{DELETE}(),\ \neg cond \curvearrowright cond\rangle]\!]\\ &\triangleq \big\langle \langle \mathsf{TranDSA}[\![\vec{a}]\!]\sigma' := \mathtt{UPDATE}(\mathsf{TranDSA}[\![\vec{a}]\!]\sigma) \curvearrowright \mathtt{DELETE}(), \mathsf{TranDSA}[\![\neg(cond)]\!]\sigma \curvearrowright\\ &\quad \mathsf{TranDSA}[\![cond]\!]\sigma\rangle, \sigma'\big\rangle;\ where\ \sigma' = \sigma[\![\vec{a} \mapsto \sigma(\vec{a}) + 1]\!]\end{aligned}$$

$$\begin{aligned}\mathsf{TranDSA}[\![\langle rs\ :=\ \mathtt{SELECT}(f(\vec{e}), r(\vec{h}(\vec{x})),\ \phi, g(\vec{e})),\ cond\rangle]\!]\sigma \triangleq\ &\big\langle \langle \mathsf{TranDSA}[\![rs]\!]\sigma' := \mathtt{SELECT}(f(\mathsf{TranDSA}[\![e]\!]\sigma), r(\vec{h}(\mathsf{TranDSA}[\![\vec{x}]\!]\sigma)), \mathsf{TranDSA}\\ &[\![\phi]\!]\sigma, g(\mathsf{TranDSA}[\![e]\!]\sigma)),\ \mathsf{TranDSA}[\![cond]\!]\sigma\rangle,\ \sigma'\big\rangle,\ where\ \sigma' = \sigma[\![\vec{a} \mapsto \sigma(\vec{a}) + 1]\!]\end{aligned}$$

$$\begin{aligned}\mathsf{TranDSA}[\![\mathtt{if}\ b\ \mathtt{then}\ stmt\ \mathtt{endif}]\!]\sigma \triangleq\ &\mathtt{if}\ \mathsf{TranDSA}[\![b]\!]\sigma\ \mathtt{then}\ \mathsf{TranDSA}[\![stmt; \mathsf{Destruct}(stmt)]\!]\sigma\ \mathtt{else}\\ &\mathsf{TranDSA}[\![\mathsf{Destruct}(stmt)]\!]\sigma\ \mathtt{endif}\end{aligned}$$

$$\begin{aligned}\mathsf{TranDSA}[\![\mathtt{if}\ b\ \mathtt{then}\ stmt_1\ \mathtt{else}\ stmt_2\ \mathtt{endif}]\!]\sigma \triangleq\ &\mathtt{if}\ \mathsf{TranDSA}[\![b]\!]\sigma\ \mathtt{then}\ \mathsf{TranDSA}[\![\mathsf{Sync}(stmt_1, stmt_2);\ \mathsf{Destruct}(stmt_1)]\!]\sigma\ \mathtt{else}\\ &\mathsf{TranDSA}[\![\mathsf{Sync}(stmt_2, stmt_1);\ \mathsf{Destruct}(stmt_2)]\!]\sigma\ \mathtt{endif}\end{aligned}$$

$$\begin{aligned}\mathsf{TranDSA}[\![stmt_1; stmt_2]\!]\sigma \triangleq\ &\langle stmt_1^{dsa}\ ;\ stmt_2^{dsa},\ \sigma''\rangle\ where, \mathsf{TranDSA}[\![stmt_1]\!]\sigma = \langle stmt_1^{dsa},\ \sigma'\rangle\\ &and\ \mathsf{TranDSA}[\![stmt_2]\!]\sigma' = \langle stmt_2^{dsa},\ \sigma''\rangle\end{aligned}$$

Figure 8: DSA Translation Function TranDSA

only DSA form whenever applicable, we restrict our discussions to DSA translation only in the subsequent sections. Let us now formalize DSA translation of the database language. Let $\sigma \in \Sigma$ be a function which maps program variables to their corresponding versions, defined as: σ: $\mathsf{Var} \rightarrow \mathbb{I}$ where $\mathbb{I}$ is the set of natural numbers. The initial version is defined by σ_0: $\mathsf{Var} \rightarrow \{0\}$. We define the function TranDSA : $\mathbb{C} \times \Sigma \rightarrow \mathbb{C}^{dsa} \times \Sigma$ for our language which maps a given command $c \in \mathbb{C}$ w.r.t. current variables-version $\sigma \in \Sigma$ into its equivalent DSA form $c^{dsa} \in \mathbb{C}^{dsa}$ resulting into a new variables-version $\sigma' \in \Sigma$. Figure 8 depicts the detailed definition of TranDSA for various components of our language under consideration. Observe that TranDSA renames variable v or attribute a into their corresponding versioned form v_i or a_i w.r.t. current version σ where $\sigma(v) = i$ or $\sigma(a) = i$. In the case of assignment statements, the current versions of defined variables are incremented by 1 resulting into an updated version σ'.

Let us now introduce a new syntactic form:

$$\langle \vec{a} := \mathsf{act}^1 \curvearrowright \mathsf{act}^2,\ cond^1 \curvearrowright cond^2\rangle$$

which indicates that the action-part act^1 on $\vec{a}$ for the tuples satisfying $cond^1$ will be followed by another action act^2 on the same $\vec{a}$ for the tuples satisfying $cond^2$. For example, `UPDATE`, `INSERT` and `DELETE` statements under this syntactic form are represented as:

$$\langle \vec{a} :=\mathtt{UPDATE}(\vec{a}) \curvearrowright \mathtt{UPDATE}(\vec{e}),\ \neg cond \curvearrowright cond\rangle$$
$$\langle \vec{a} :=\mathtt{UPDATE}(\vec{a}) \curvearrowright \mathtt{INSERT}(\vec{e}),\ true \curvearrowright false\rangle$$

$\langle \vec{a} := \texttt{UPDATE}(\vec{a}) \curvearrowright \texttt{DELETE}(),\ \neg cond \curvearrowright cond \rangle$

Since the DSA forms of UPDATE, DELETE, and INSERT add an auxiliary UPDATE statement before them in order to capture the definition of attributes for all tuples, we use this syntactic form in the definition of TranDSA. Observe that, like assignment statements, in these cases also σ is modified to σ' by increasing the versions of defined attributes by 1. In the case of conditional statements if- and if-else, we use two functions Destruct and Sync, defined below:

$\text{Destruct}(v := e) \triangleq v := v$

$\text{Destruct}(\langle \vec{a} := \texttt{UPDATE}(\vec{e}),\ cond \rangle) \triangleq \langle \vec{a} := \texttt{UPDATE}(\vec{a}),\ true \rangle$

$\text{Destruct}(\langle \vec{a} := \langle \texttt{INSERT}(\vec{e}), false \rangle) \triangleq \langle \vec{a} := \langle \texttt{UPDATE}(\vec{a}), true \rangle$

$\text{Destruct}(\langle \vec{a} := \texttt{DELETE}(),\ cond \rangle) \triangleq \langle \vec{a} := \texttt{UPDATE}(\vec{a}),\ true \rangle$

For Other Statements $stmt$: $\text{Destruct}(stmt) \triangleq skip$

$\text{Destruct}(stmt_1;\ stmt_2) \triangleq \text{Destruct}(stmt_1);\ \text{Destruct}(stmt_2)$

$\text{Sync}(stmt_i, stmt_j) \triangleq stmt_i; \text{Destruct}(stmt_j - stmt_i)$

where ($stmt_j$ - $stmt_i$) represents only those defining statements in $stmt_j$ which are not common to both $stmt_i$ and $stmt_j$. Note that the task of the Sync function is to make variables definitions in both if- and else-blocks consistent by adding a new part ($stmt_j$ - $stmt_i$) only after applying Destruct on it ensuring that no change in the values of the variables takes place before and after their inclusion. For example, given the following if-else code snippet "if $(x \geqslant 0)$ then ① UPDATE T SET $a := a + 10$ WHERE $c > 10$ AND $c < 20$; else ② UPDATE T SET $b := b$ - 15 WHERE $c > 25$ AND $c < 35$; endif". According to the definition of TranDSA in case of if-else, the use of Sync and Destruct functions yields the following DSA form:

if $x_0 > 0$ then

① UPDATE $\hat{\text{T}}$ SET $a_1 = a_0 + 10$ WHERE $c > 10$ AND $c < 20$;
UPDATE $\hat{\text{T}}$ SET $a_1 = a_0$ WHERE $\neg(c > 10$ AND $c < 20)$;

①a UPDATE $\hat{\text{T}}$ SET $b_2 = b_0$;

①b UPDATE $\hat{\text{T}}$ SET $a_2 = a_1$;

else

② UPDATE $\hat{\text{T}}$ SET $b_1 = b_0 - 15$ WHERE $c > 25$ AND $c < 35$;
UPDATE $\hat{\text{T}}$ SET $b_1 = b_0$ WHERE $\neg(c > 25$ AND $c < 35)$;

②a UPDATE $\hat{\text{T}}$ SET $a_2 = a_0$;

②b UPDATE $\hat{\text{T}}$ SET $b_2 = b_1$;

endif

Observe that Sync(①, ②) introduces statement ①a under the if-block of the resultant DSA form, followed by statement ①b which is obtained by Destruct(①). Similarly, statements ②a and ②b are introduced under else-block.

In general, TranDSA can be implemented based on the standard DSA construction algorithm [37] with an extension to cover SQL statements.

We are now in a position to propose Verification Condition Generation techniques, namely symbolic execution, conditional normal form, and weakest precondition, in the subsequent section.

3. Verification Condition Generation Techniques

As we mentioned earlier that the fundamental step involved in the deductive-based approach is to generate VCs from program codes annotated with specifications. Once VCs are generated, they are fed to the theorem provers to check their validity which proves the correctness of the programs w.r.t. given specifications. Although VCs have been widely used in some well-known verification tools [38, 39, 40], they have not been systematically analysed by the research community. Authors in [13, 29, 30] first revisited various VC generation techniques for iteration free imperative programs and performed a detailed comparison in term of efficiency. To the best of our knowledge, this has never been explored in the realm of database applications where database statements are embedded within a general purpose host imperative language. Our main objective in this section is to extend these VC generation techniques, namely symbolic execution, conditional normal form and weakest precondition, to the case of database applications, enabling to formally verify underlying database properties and to perform a detailed comparative analysis with respect to performance.

3.1. Symbolic Execution

The very first and simplest method to verify the correctness of a program is to generate logical formulas along all execution paths of a program using symbolic execution, taking the given specification into account. Symbolic execution considers symbolic input values (rather than concrete values) and therefore execution proceeds along all control paths covering the entire execution space of the programs. Notice that the method produces one VC for each *assert* command in a path and each VC encodes only the part of the program that is relevant for that assert command. Therefore, there exists a one-to-one relation between the execution path and the VC. the validity of each logical formula certifies that execution going through the corresponding path meets the assertions, and consequently, successful validation of all formulas ensures the program's correctness w.r.t. the given specification. This technique has an advantage from the point of view of *traceability*: an invalid VC immediately identifies the executions that may violate a property.

Let us now formalize below two functions Path and VC^{se}. Note that, in order to indicate a statement in DSA form, we use either the superscript dsa or the subscripts i and $i+1$ as the current version and the updated version respectively. We denote by $\langle \vec{a}_{i+1} := \text{act}_i^1 \curvearrowright \text{act}_i^2,\ cond_i^1 \curvearrowright cond_i^2 \rangle$ the DSA form of either UPDATE or DELETE or INSERT as follows:

$\langle \vec{a}_{i+1} := \texttt{UPDATE}(\vec{a}_i) \curvearrowright \texttt{UPDATE}(\vec{e}_i),\ \neg cond_i \curvearrowright cond_i \rangle$

$\langle \vec{a}_{i+1} := \texttt{UPDATE}(\vec{a}_i) \curvearrowright \texttt{INSERT}(\vec{e}_i),\ true \curvearrowright false \rangle$

$\langle \vec{a}_{i+1} := \texttt{UPDATE}(\vec{a}_i) \curvearrowright \texttt{DELETE}(),\ \neg cond_i \curvearrowright cond_i \rangle$

The functions Path and VC^{se} are defined below:

(1) Let Φ be a logical encoding of an execution path π up

$$\mathsf{Path}(\Phi,\ \texttt{skip}) = \Phi \qquad \mathsf{Path}(\Phi,\ \texttt{assume}\ \phi^{dsa}) = \Phi\ \wedge\ \phi^{dsa} \qquad \mathsf{Path}(\Phi,\ v_{i+1} := e_i) = \Phi\ \wedge\ v_{i+1} = e_i \qquad \mathsf{Path}(\Phi,\ \texttt{assert}\ \phi^{dsa}) = \Phi\ \wedge\ \phi^{dsa}$$

$$\mathsf{Path}(\Phi,\ \langle rs_{i+1} := \texttt{SELECT}(f(\vec{e}_i), r(\vec{h}(\vec{a}_i)),\ \phi',\ g(\vec{e}_i)),\ cond_i\rangle) = \Phi \wedge ((cond_i \wedge rs_{i+1} = \mathsf{F}(\vec{a}_i)) \vee (\neg cond_i \wedge\ rs_{i+1} = rs_i))$$

$$\mathsf{Path}(\Phi,\ \langle \vec{a}_{i+1} := \mathsf{act}_i^1 \curvearrowright \mathsf{act}_i^2,\ cond_i^1 \curvearrowright cond_i^2\rangle) = \mathsf{Path}(\Phi, \langle \vec{a}_{i+1} := \mathsf{act}_i^1,\ cond_i^1\rangle) \vee \mathsf{Path}(\Phi, \langle \vec{a}_{i+1} := \mathsf{act}_i^2,\ cond_i^2\rangle)$$

$$\mathsf{Path}(\Phi,\ \langle \vec{a}_{i+1} := \texttt{UPDATE}(\vec{e}_i),\ cond_i\rangle) = \Phi \wedge (cond_i \wedge \bigwedge_{j=1}^{|\vec{a}_{i+1}|} a_{i+1}^j = e_i^j) \qquad \mathsf{Path}(\Phi,\ \langle \vec{a}_{i+1} := \texttt{INSERT}(\vec{e}_i),\ false\rangle) = \Phi\ \wedge\ (\bigwedge_{j=1}^{|\vec{a}_{i+1}|} a_{i+1}^j = e_i^j)$$

$$\mathsf{Path}(\Phi,\ \langle \vec{a}_{i+1} := \texttt{DELETE}(),\ cond_i\rangle) = \Phi \wedge (\neg cond_i \wedge \bigwedge_{j=1}^{|\vec{a}_{i+1}|} a_{i+1}^j = a_i^j) \qquad \mathsf{Path}(\Phi,\ stmt_1^{dsa};\ stmt_2^{dsa}) = \mathsf{Path}(\mathsf{Path}(\Phi,\ stmt_1^{dsa}),\ stmt_2^{dsa})$$

$$\mathsf{Path}(\Phi,\ \texttt{if}\ b_i\ \texttt{then}\ stmt_1^{dsa}\ \texttt{else}\ stmt_2^{dsa}) = \mathsf{Path}(\Phi \wedge b_i,\ stmt_1^{dsa}) \cup \mathsf{Path}(\Phi \wedge \neg b_i,\ stmt_2^{dsa})$$

Figure 9: Encoding of database statements into logical formula

to program point ℓ. On encountering $stmt^{dsa}$ at program point $\ell + 1$ along the path, the function Path(Φ, $stmt^{dsa}$) returns a logical formula that encodes π up to the program point $\ell + 1$. Figure 9 defines the function Path for all statements in our language. Observe that the helper function F(.) represents the composition of functions g, ϕ, h, r, f (if present) in SELECT.

(2) Given a program in the form of a sequence of statements, the function VC^{se} recursively collects the logical encoding of all program paths by invoking the auxiliary function Path defined above. Finally, on encountering an *assert* statement, the function VC^{se} returns VC with an implication to the *assert* constraints. This is defined in Figure 10.

$$\mathsf{VC}^{se}(\Phi,\ \mathtt{stmt}_1^{dsa};\ \mathtt{stmt}_2^{dsa}) = \mathsf{VC}^{se}(\Phi, \mathtt{stmt}_1^{dsa}) \cup \mathsf{VC}^{se}(\mathsf{Path}(\Phi, \mathtt{stmt}_1^{dsa}), \mathtt{stmt}_2^{dsa})$$

$$\mathsf{VC}^{se}(\Phi,\ \texttt{if}\ b\ \texttt{then}\ \mathtt{stmt}_1^{dsa}\ \texttt{else}\ \mathtt{stmt}_2^{dsa}) = \mathsf{VC}^{se}(\Phi \wedge \mathsf{b}, \mathtt{stmt}_1^{dsa}) \cup \mathsf{VC}^{se}(\Phi \wedge \neg\mathsf{b}, \mathtt{stmt}_2^{dsa})$$

$$\mathsf{VC}^{se}(\Phi,\ \texttt{assert}\ \phi^{dsa}) = \Phi \Rightarrow \phi^{dsa}$$

$$\mathsf{VC}^{se}(\Phi,\ \texttt{assume}\ \phi^{dsa}) = \emptyset$$

$$\mathsf{VC}^{se}(\Phi,\ v_{i+1} := e_i) = \emptyset \qquad \mathsf{VC}^{se}(\Phi,\ \texttt{skip}) = \emptyset$$

$$\mathsf{VC}^{se}(\Phi,\ Q_{sel}^{dsa}) = \emptyset \qquad \mathsf{VC}^{se}(\Phi,\ Q_{upd}^{dsa}) = \emptyset$$

$$\mathsf{VC}^{se}(\Phi,\ Q_{ins}^{dsa}) = \emptyset \qquad \mathsf{VC}^{se}(\Phi,\ Q_{del}^{dsa}) = \emptyset$$

Figure 10: VC Generation using symbolic execution

The overall algorithm of symbolic execution-based VC generation is depicted in Algorithm 1. CFG($\mathcal{P}^{dsa}$) denotes control-flow graph of $\mathcal{P}^{dsa}$.
Let us now illustrate the function VC^{se} on our motivating example in Figure 1.

Example 1. *Given the code snippet* DBprog *in Figure 1, its DSA form annotated with assume and assert statements is shown in Figure 11. Initially* Φ *is the empty set* $\emptyset$*. Consider the sequence of statements* $\langle 6;\ 7;\ 8;\ \ldots\rangle$ *in* DBprog^{dsa}*. According to the definition of* VC^{se}*, applying Algorithm 1 on the sequence of statements,* $\mathsf{VC}^{se}(\Phi, \langle 6;\ 7;\ 8;\ \ldots\rangle)$ *results*

Algorithm 1: SE-based VCG

Input: DSA form $\mathcal{P}^{dsa}$ of annotated database program $\mathcal{P}$
Output: A set of VCs

```
1 X := ∅;
2 for each path n_1 n_2 ... n_l ∈ CFG(P^dsa) do
3     Φ := ∅;
      // Let S = s_1; s_2; ...; s_n; where s_i is the
         statement in P^dsa corresponding to
         node n_i
4     X := X ∪ VC^se(Φ, S);
5 Return X;
6 End
```

in $\mathsf{VC}^{se}(\Phi, 6) \cup \mathsf{VC}^{se}(\mathsf{Path}(\Phi, \langle 6\rangle), \langle 7; 8; \ldots\rangle)$ *which initiates recursive calls on the subproblems. The auxiliary function* Path *accumulates logical formulas along the program paths, and finally the function* VC^{se}*, on encountering the assert statement at program point 19, generates the following set of* VCs*.*

$$\begin{aligned}\mathsf{VC}_1:\ &((\mathrm{MP}_0 \geqslant 80000 \wedge \mathrm{EQ}_0 \geqslant 60000 \wedge \mathrm{CT}_0 \geqslant 10000 \wedge \mathrm{CS}_0 \geqslant 5000)\\ &\wedge ((z_1 == \mathrm{TA}_0 \wedge \mathrm{Did}_0 == y_0) \vee (z_1 == z_0 \wedge \neg(\mathrm{Did}_0 == y_0))) \wedge\\ &(z_1 \geqslant x_0 \wedge m_1 == z_1 - x_0 \wedge n_1 == (m_1/4)) \wedge ((\mathrm{MP}_1 == \mathrm{MP}_0 - n_1\\ &\wedge \mathrm{EQ}_1 == \mathrm{EQ}_0 - n_1 \wedge \mathrm{CT}_1 == \mathrm{CT}_0 - n_1 \wedge \mathrm{CS}_1 == \mathrm{CS}_0 - n_1\\ &\wedge \mathrm{Did}_1 == x_0) \vee (\mathrm{MP}_1 == \mathrm{MP}_0 \wedge \mathrm{EQ}_1 == \mathrm{EQ}_0 \wedge \mathrm{CT}_1 == \mathrm{CT}_0\\ &\wedge \mathrm{CS}_1 == \mathrm{CS}_0 \wedge \neg(\mathrm{Did}_1 == x_0))) \wedge (m_2 == m_1 \wedge n_2 == n_1) \wedge\\ &(\mathrm{MP}_2 == \mathrm{MP}_1 \wedge \mathrm{EQ}_2 == \mathrm{EQ}_1 \wedge \mathrm{CT}_2 == \mathrm{CT}_1 \wedge \mathrm{CS}_2 == \mathrm{CS}_1))\\ &\Rightarrow (\mathrm{MP}_2 \geqslant 80000 \wedge \mathrm{EQ}_2 \geqslant 60000 \wedge \mathrm{CT}_2 \geqslant 10000 \wedge \mathrm{CS}_2 \geqslant 5000)\end{aligned}$$

$$\begin{aligned}\mathsf{VC}_2:\ &((\mathrm{MP}_0 \geqslant 80000 \wedge \mathrm{EQ}_0 \geqslant 60000 \wedge \mathrm{CT}_0 \geqslant 10000 \wedge \mathrm{CS}_0 \geqslant 5000) \wedge\\ &((z_1 == \mathrm{TA}_0 \wedge \mathrm{Did}_0 == y_0) \vee (z_1 == z_0 \wedge \neg(\mathrm{Did}_0 == y_0))) \wedge\\ &\neg(z_1 \geqslant x_0) \wedge \mathrm{MP}_2 == \mathrm{MP}_0 \wedge \mathrm{EQ}_2 == \mathrm{EQ}_0 \wedge \mathrm{CT}_2 == \mathrm{CT}_0 \wedge\\ &\mathrm{CS}_2 == \mathrm{CS}_0 \wedge m_2 == m_0 \wedge n_2 == n_0) \Rightarrow (\mathrm{MP}_2 \geqslant 80000 \wedge\\ &\mathrm{EQ}_2 \geqslant 60000 \wedge \mathrm{CT}_2 \geqslant 10000 \wedge \mathrm{CS}_2 \geqslant 5000)\end{aligned}$$

The invalidity of VC_1 *in some cases (for example,* z_1 *= 20,* x_0 *= 12,* MP_2 *= 79999,* EQ_2 *= 59999,* CT_2 *= 9999,* CS_2 *= 4999) indicates that* DBprog *does not respect the given specification all the time. In particular, the violation happens due to statement 10 in* DBprog *where attribute values*

are subtracted by some amount which is influenced by a run-time input.

```
1.  CREATE OR REPLACE PROCEDURE DBprog (y0 int, x0 int)
      IS
2.  z0 int;
3.  m0 int;
4.  n0 int;
5.  BEGIN
6.    assume MP0 ⩾ 80000 and EQ0 ⩾ 60000 and CT0 ⩾ 10000 and
       CS0 ⩾ 5000;
7.    SELECT TA0 INTO z1 FROM Budget WHERE Did0= y0;
8.    if z1 ⩾ x0 then
9.        m1 := z1 − x0;
10.       n1 := m1/4;
11.       UPDATE Budget SET MP1 = MP0−n1, EQ1 = EQ0−n1,
          CT1 = CT0−n1, CS1 = CS−n1 WHERE Did0 = x0;
          UPDATE Budget SET MP1 = MP0, EQ1 = EQ0, CT1 =
           CT0, CS1 = CS0 WHERE ¬(Did0 = x0);
12.       m2 := m1;
13.       n2 := n1;
14.       UPDATE Budget SET MP2 = MP1, EQ2 = EQ1,
          CT2 = CT1, CS2 = CS1;
      else
15.       UPDATE Budget SET MP2 = MP0, EQ2 = EQ0,
          CT2 = CT0, CS2 = CS0;
16.       m2 := m0;
17.       n2 := n0;
18.   endif;
19.   assert MP2 ⩾ 80000 and EQ2 ⩾ 60000 and CT2 ⩾ 10000 and
       CS2 ⩾ 5000;
20. end;
```

Figure 11: DBprog^{dsa}: DSA form of DBprog.

Limitation: The major problem of symbolic execution-based VCG is that the number of VCs will be exponential in the worst case scenario. For instance, a program with n conditional statements may result in $O(2^n)$ number of VCs.

3.2. Conditional Normal Form

We have seen in the previous section that VCG using symbolic execution leads to an exponential number of VCs in the worst case scenario. To overcome this problem, we now describe the second method of VCG which was first used in Bounded Model Checking of software (BMC) [39] as a way to unroll loops avoiding path enumerations. In the BMC technique, the transformation of iteration free DSA programs form is basically guided by the following three rules [39]:

$$R_1: \texttt{if}\ (b^{dsa})\ stmt_1^{dsa}\ \texttt{else}\ stmt_2^{dsa} \Rightarrow \texttt{if}\ (b^{dsa})\ stmt_1^{dsa}; \texttt{if}\ (\neg b^{dsa})\ stmt_2^{dsa};$$

$$R_2: \texttt{if}(b^{dsa})\ \{stmt_1^{dsa};\ stmt_2^{dsa}\} \Rightarrow \texttt{if}\ (b^{dsa})\ stmt_1^{dsa};\ \texttt{if}\ (b^{dsa})\ stmt_2^{dsa};$$

$$R_3: \texttt{if}\ (b_1^{dsa})\{\texttt{if}\ (b_2^{dsa})\ stmt^{dsa}\} \Rightarrow \texttt{if}\ (b_1^{dsa} \wedge b_2^{dsa})\ stmt^{dsa}$$

The first rule R_1 states that branches of conditional statements can be sequentialized. The second rule R_2 states that conditions can be distributed through the sequence of statements present in the body of conditional statements. The third rule R_3 states that nested conditions can be combined together. Observe that the above rules rewrite a given DSA program into another semantically equivalent form where every statement $stmt^{dsa}$ is guarded by a condition b^{dsa}. If a $stmt^{dsa}$ is not in the body of any conditional statement then it will be guarded by *true*. This representation of the DSA programs is known as Conditional Normal Form (CNF).

Let us now define the CNF-based VCG for our database language by following the below steps:

(1) Transformation of database program $\mathcal{P}^{dsa}$ in DSA form into its equivalent CNF form $\mathcal{P}^{cnf}$. This is defined by function toCNF(.): $\mathbb{C}^{dsa} \times \Upsilon \rightarrow \mathbb{C}^{cnf}$, which transforms a given statement $c^{dsa} \in \mathbb{C}^{dsa}$ into its equivalent CNF form $c^{cnf} \in \mathbb{C}^{cnf}$ under the path condition $\rho \in \Upsilon$ upon which the execution of c^{dsa} depends.

$$\begin{aligned}
\text{toCNF}(\rho,\ \texttt{skip}) &= \texttt{if}\ \rho\ \texttt{then skip}\\
\text{toCNF}(\rho,\ v_i := e_i) &= \texttt{if}\ \rho\ \texttt{then}\ v_i := e_i\\
\text{toCNF}(\rho,\ assume\ \phi^{dsa}) &= \texttt{if}\ \rho\ \texttt{then}\ assume\ \phi^{dsa}\\
\text{toCNF}(\rho,\ Q_{sel}^{dsa}) &= \texttt{if}\ \rho\ \texttt{then}\ Q_{sel}^{dsa}\\
\text{toCNF}(\rho,\ Q_{ins}^{dsa}) &= \texttt{if}\ \rho\ \texttt{then}\ Q_{ins}^{dsa}\\
\text{toCNF}(\rho,\ Q_{upd}^{dsa}) &= \texttt{if}\ \rho\ \texttt{then}\ Q_{upd}^{dsa}\\
\text{toCNF}(\rho,\ Q_{del}^{dsa}) &= \texttt{if}\ \rho\ \texttt{then}\ Q_{del}^{dsa}\\
\text{toCNF}(\rho,\ stmt_1^{dsa};\ stmt_2^{dsa}) &= \text{toCNF}(\rho,\ stmt_1^{dsa});\\
&\quad\ \text{toCNF}(\rho,\ stmt_2^{dsa})\\
\text{toCNF}(\rho,\ \texttt{if}\ b\ \texttt{then}\ stmt_1^{dsa}&\\
\texttt{else}\ stmt_2^{dsa}\texttt{endif}) &= \text{toCNF}(\rho \wedge b,\ stmt_1^{dsa});\\
&\quad\ \text{toCNF}(\rho \wedge \neg b,\ stmt_2^{dsa})\\
\text{toCNF}(\rho,\ assert\ \phi^{dsa}) &= \texttt{if}\ \rho\ \texttt{then}\ assert\ \phi^{dsa}
\end{aligned}$$

(2) Extraction of two sets of formulas f_{stmt} and f_{pr} from each statement in $\mathcal{P}^{cnf}$. To this purpose, we define the function Conf(.): $\mathbb{C}^{cnf} \rightarrow \mathbb{W} \times \mathbb{W}$, where $\mathbb{W}$ is the set of well-formed first order logic formulas and $\mathsf{f}_{stmt}, \mathsf{f}_{pr} \in \mathbb{W}$. This is depicted in Figure 12. Finally, a single verification condition is constructed in the form of $\bigwedge \mathsf{f}_{stmt} \Rightarrow \bigwedge \mathsf{f}_{pr}$, where f_{stmt} contains the encoding of program statements and f_{pr} contains the encoding of all *assert* commands in the program. Note that, unlike symbolic execution, in this technique only one verification condition is generated.

Figure 13 depicts the function VC^{cnf} which combines these two steps and generates a VC from a given program in DSA form. Algorithm 2 is the overall algorithm to generate the CNF-based VC. Example 2 illustrates the CNF-based VCG technique.

Algorithm 2: CNF-based VCG

Input: DSA form $\mathcal{P}^{dsa}$ of annotated database program $\mathcal{P}$
Output: Single VC

1 $\rho := true$;
2 Let $\mathcal{P}^{dsa}$ be a sequence of statements $s_1; s_2; \ldots; s_n$;
3 $(\mathsf{f}_{stmt}, \mathsf{f}_{pr}) := \text{VC}^{cnf}(\rho, \mathcal{P}^{dsa})$
4 VC := $(\bigwedge \mathsf{f}_{stmt} \Rightarrow \bigwedge \mathsf{f}_{pr})$
5 Return VC
6 **End**

Example 2. *Given the DSA form of* DBprog *in Figure 11, its* CNF *form applying* ToCNF(.) *is depicted in Figure 14. Figure 15(a) shows the output formula set* f_{stmt} *and* f_{pr} *obtained by applying* Conf(.)*. Therefore, the* VC *is constructed*

$$\mathsf{Conf}(\texttt{if } b_i \texttt{ then } \langle rs_{i+1} := \mathsf{SELECT}(f(\vec{e}), r(\vec{h}(\vec{x})), \phi, g(\vec{e})), cond_i\rangle) = (\{(b_i \wedge cond_i \Rightarrow rs_{i+1} = \mathsf{F}(\vec{a}_i)) \vee (b_i \wedge \neg cond_i \Rightarrow rs_{i+1} = rs_i)\}, \emptyset)$$

$$\mathsf{Conf}(\langle \vec{a}_{i+1} := \mathsf{act}_i^1 \curvearrowright \mathsf{act}_i^2,\ cond_i^1 \curvearrowright cond_i^2\rangle) = \mathsf{Conf}(\langle \vec{a}_{i+1} := \mathsf{act}_i^1,\ cond_i^1\rangle) \vee \mathsf{Conf}(\langle \vec{a}_{i+1} := \mathsf{act}_i^2,\ cond_i^2\rangle) \qquad \mathsf{Conf}(\texttt{if } b_i \texttt{ then skip}) = (\emptyset,\ \emptyset)$$

$$\mathsf{Conf}(\texttt{if } b_i \texttt{ then } \langle \vec{a}_{i+1} := \mathsf{UPDATE}(\vec{e}_i),\ cond_i\rangle) = (\{(b \wedge cond_i \Rightarrow \bigwedge_{i=1}^{|\vec{a}_i|}(\vec{a}_{i+1} = \vec{e}_i))\},\ \emptyset) \qquad \mathsf{Conf}(\texttt{if } b_i \texttt{ then } v_i := e_i) = (\{b_i \Rightarrow v_i = e_i\},\ \emptyset)$$

$$\mathsf{Conf}(\texttt{if } b_i \texttt{ then } \langle \vec{a}_{i+1} := \mathsf{INSERT}(\vec{e}_i), false\rangle) = (\{b \Rightarrow (\bigwedge_{i=1}^{|\vec{a}_i|}(\vec{a}_{i+1} = \vec{e}_i))\},\ \emptyset) \qquad \mathsf{Conf}(\texttt{if } b_i \texttt{ then } \textit{assume } \phi_i) = (\{b_i \Rightarrow \phi_i\},\ \emptyset)$$

$$\mathsf{Conf}(\texttt{if } b_i \texttt{ then } \langle \vec{a}_{i+1} := \mathsf{DELETE}(),\ cond_i\rangle) = (\{\rho \wedge \neg cond_i \Rightarrow \bigwedge_{i=1}^{|\vec{a}_i|}(\vec{a}_{i+1} = \vec{a}_i))\},\ \emptyset) \qquad \mathsf{Conf}(\texttt{if } b_i \texttt{ then } \textit{assert } \phi_i) = (\emptyset,\ \{b_i \Rightarrow \phi_i\})$$

$$\mathsf{Conf}(stmt_1^{dsa};\ stmt_2^{dsa}) = (\mathsf{f}_{stmt_1} \cup \mathsf{f}_{stmt_2},\ \mathsf{f}_{pr_1} \cup \mathsf{f}_{pr_2}),\ \textit{where } \mathsf{Conf}(stmt_1^{dsa}) = (\mathsf{f}_{stmt_1},\ \mathsf{f}_{pr_1}),\ \mathsf{Conf}(stmt_2^{dsa}) = (\mathsf{f}_{stmt_2},\ \mathsf{f}_{pr2})$$

Figure 12: Function to compute $\bigwedge \mathsf{f}_{stmt}$ and $\bigwedge \mathsf{f}_{pr}$

$$\mathsf{VC}^{cnf}(\rho,\ \texttt{skip}) = (\emptyset,\ \emptyset) \qquad \mathsf{VC}^{cnf}(\rho,\ v_i := e_i) = (\{\rho \Rightarrow v_i = e_i\},\ \emptyset) \qquad \mathsf{VC}^{cnf}(\rho,\ \textit{assume } \phi_i) = (\{\rho \Rightarrow \phi_i\},\ \emptyset)$$

$$\mathsf{VC}^{cnf}(\rho,\ \langle rs_{i+1} := \mathsf{SELECT}(f(\vec{e}), r(\vec{h}(\vec{x})), \phi, g(\vec{e})), cond_i\rangle) = (\{(\rho \wedge cond_i \Rightarrow rs_{i+1} = \mathsf{F}(\vec{a}_i)) \ \vee \ (\rho \wedge \neg cond_i \Rightarrow rs_{i+1} = rs_i)\}, \emptyset)$$

$$\mathsf{VC}^{cnf}(\rho,\ \langle \vec{a}_{i+1} := \mathsf{act}_i^1 \curvearrowright \mathsf{act}_i^2,\ cond_i^1 \curvearrowright cond_i^2\rangle) = \mathsf{VC}^{cnf}(\rho,\ \langle \vec{a}_{i+1} := \mathsf{act}_i^1,\ cond_i^1\rangle) \ \vee \ \mathsf{VC}^{cnf}(\rho,\ \langle \vec{a}_{i+1} := \mathsf{act}_i^2,\ cond_i^2\rangle)$$

$$\mathsf{VC}^{cnf}(\rho, \langle \vec{a}_{i+1} := \mathsf{UPDATE}(\vec{e}_i),\ cond_i\rangle) = (\{(\rho \wedge cond_i \Rightarrow \bigwedge_{i=1}^{|\vec{a}_i|}(\vec{a}_{i+1} = \vec{e}_i))\},\ \emptyset) \qquad \mathsf{VC}^{cnf}(\rho,\ \langle \vec{a}_{i+1} := \mathsf{INSERT}(\vec{e}_i), f\,alse\rangle) = (\{\rho \Rightarrow (\bigwedge_{i=1}^{|\vec{a}_i|}(\vec{a}_{i+1} = \vec{e}_i))\},\ \emptyset)$$

$$\mathsf{VC}^{cnf}(\rho,\ \langle \vec{a}_{i+1} := \mathsf{DELETE}(),\ cond_i\rangle) = (\{\rho \wedge \neg cond_i \Rightarrow \bigwedge_{i=1}^{|\vec{a}_i|}(\vec{a}_{i+1} = \vec{a}_i))\},\ \emptyset) \qquad \mathsf{VC}^{cnf}(\rho,\ \textit{assert } \phi_i) = (\emptyset,\ \{\rho \ \Rightarrow \phi_i\})$$

$$\mathsf{VC}^{cnf}(\rho,\ stmt_1^{dsa};\ stmt_2^{dsa}) = (\mathsf{f}_{stmt_1} \cup \mathsf{f}_{stmt_2},\ \mathsf{f}_{pr1} \cup \mathsf{f}_{pr2}),\ \textit{where } (\mathsf{f}_{stmt_1},\ \mathsf{f}_{pr1}) = \mathsf{VC}^{cnf}(\rho,\ stmt_1^{dsa}),\ (\mathsf{f}_{stmt_2},\ \mathsf{f}_{pr2}) = \mathsf{VC}^{cnf}(\rho,\ stmt_2^{dsa})$$

$$\mathsf{VC}^{cnf}(\rho,\ \texttt{if } b_i \texttt{ then } stmt_1^{dsa} \texttt{ else } stmt_2^{dsa} \texttt{ endif}) = (\mathsf{f}_{stmt_1} \cup \mathsf{f}_{stmt_2},\ \mathsf{f}_{pr1} \cup \mathsf{f}_{pr2}),\ \textit{where } (\mathsf{f}_{stmt_1},\ \mathsf{f}_{pr1}) = \mathsf{VC}^{cnf}(\rho \wedge b_i,\ stmt_1^{dsa}),\ (\mathsf{f}_{stmt_2},\ \mathsf{f}_{pr2}) = \mathsf{VC}^{cnf}(\rho \wedge \neg b_i,\ stmt_2^{dsa})$$

Figure 13: Function to generate CNF-based VC

by combining f_{stmt} *and* f_{pr} *in the form* $\bigwedge \mathsf{f}_{stmt} \Rightarrow \bigwedge \mathsf{f}_{pr}$ *is depicted in Figure 15(b).*

```
1.  CREATE OR REPLACE PROCEDURE Proc_Budget_Adjust
      (y0 int, x0 int) IS
                ⋮
5.  BEGIN
6.     if true then
7.        assume MP0 ⩾ 80000 and EQ0 ⩾ 60000 and CT0 ⩾ 10000
       and CS0 ⩾ 5000;
9.     endif
7.     if true then
8.        SELECT TA0 INTO z1 FROM Budget WHERE Did0= y0;
9.     endif
10.    if z1 ⩾ x0 then
11.       m1 := z1 − x0;
12.    endif
                ⋮
38.    if true then
39.       assert MP2 ⩾ 80000 and EQ2 ⩾ 60000 and CT2 ⩾ 10000 and
       CS2 ⩾ 5000;
40.    endif
41. end;
```

Figure 14: DBprog^{cnf}: CNF form of DBprog^{dsa}.

Observe that, similarly to when the symbolic execution technique was used, the validity checking of VC^{cnf} *yields invalidity for some cases which indicate a violation of the property* $\mathrm{MP} \geqslant 80000 \wedge \mathrm{EQ} \geqslant 60000 \wedge \mathrm{CT} \geqslant 10000 \wedge \mathrm{CS} \geqslant 5000$.

3.3. Weakest Precondition

Hoare logic is a widely-used deductive verification formalism for computer programs [41]. The axioms and inference rules of this proof system are based on Hoare triples of the form $\{\mathtt{Pre}\}stmt\{\mathtt{Post}\}$. This means any terminating execution of the statement *stmt* on a state satisfying the precondition `Pre` results in a new state satisfying the postcondition `Post`.

Dijkstra [42] introduced predicate transformers, a set of rules for transforming predicates on states, as a way to specify program semantics. In particular, he defined "Weakest Precondition (wp)" and "Strongest Postcondition (sp)", treating assertions like preconditions and postconditions as predicates on program states. The predicate transformers technique generates VCs by propagating predicates either backward (weakest preconditions) or forward (strongest postconditions) along the program. As VCG based on the strongest postcondition is costly due to the presence of quan-

f_{stmt} : { $(true \Rightarrow MP_0 \geqslant 80000 \wedge EQ_0 \geqslant 60000 \wedge CT_0 \geqslant 10000 \wedge CS_0 \geqslant 5000)$
$(true \Rightarrow ((z_1 == TA_0 \wedge Did_0 == y_0) \vee (z_1 == z_0 \wedge \neg(Did_0 == y_0))))$,
$(z_1 \geqslant x_0 \Rightarrow m_1 == z_1 - x_0)$,
$(z_1 \geqslant x_0 \Rightarrow n_1 == m_1/4)$,
$((z_1 \geqslant x_0 \wedge Did_1 == x_0 \Rightarrow (MP_1 == MP_0 - n_1 \wedge EQ_1 == EQ_0 - n_1 \wedge CT_1 == CT_0 - n_1 \wedge CS_1 == CS_0 - n_1)) \vee (z_1 \geqslant x_0 \wedge \neg(Did_1 == x_0) \Rightarrow (MP_1 == MP_0 \wedge EQ_1 == EQ_0 \wedge CT_1 == CT_0 \wedge CS_1 == CS_0))$,
$(z_1 \geqslant x_0 \Rightarrow m_2 == m_1)$,
$(z_1 \geqslant x_0 \Rightarrow n_2 == n_1)$,
$(z_1 \geqslant x_0 \Rightarrow MP_2 == MP_1 \wedge EQ_2 == EQ_1 \wedge CT_2 == CT_1 \wedge CS_2 == CS_1)$,
$(\neg(z_1 \geqslant x_0) \Rightarrow MP_2 == MP_0 \wedge EQ_2 == EQ_0 \wedge CT_2 == CT_0 \wedge CS_2 == CS_0)$,
$(\neg(z_1 \geqslant x_0) \Rightarrow m_2 == m_0)$,
$(\neg(z_1 \geqslant x_0) \Rightarrow n_2 == n_0)$}
f_{pr} : { $(true \Rightarrow MP_2 \geqslant 80000 \wedge EQ_2 \geqslant 60000 \wedge CT_2 \geqslant 10000 \wedge CS_2 \geqslant 5000)$}

(a) Formula sets f_{stmt} and f_{pr} from DBprog^{cnf} by applying Conf(.)

VC : $((true \Rightarrow MP_0 \geqslant 80000 \wedge EQ_0 \geqslant 60000 \wedge CT_0 \geqslant 10000 \wedge CS_0 \geqslant 5000) \wedge (true \Rightarrow ((z_1 == TA_0 \wedge Did_0 == y_0) \vee (z_1 == z_0 \wedge \neg(Did_0 == y_0)))) \wedge (z_1 \geqslant x_0 \Rightarrow m_1 == z_1 - x_0) \wedge (z_1 \geqslant x_0 \Rightarrow n_1 == m_1/4) \wedge ((z_1 \geqslant x_0 \wedge Did_1 == x_0 \Rightarrow (MP_1 == MP_0 - n_1 \wedge EQ_1 == EQ_0 - n_1 \wedge CT_1 == CT_0 - n_1 \wedge CS_1 == CS_0 - n_1)) \vee (z_1 \geqslant x_0 \wedge \neg(Did_1 == x_0) \Rightarrow (MP_1 == MP_0 \wedge EQ_1 == EQ_0 \wedge CT_1 == CT_0 \wedge CS_1 == CS_0)) \wedge (z_1 \geqslant x_0 \Rightarrow m_2 == m_1) \wedge (z_1 \geqslant x_0 \Rightarrow n_2 == n_1) \wedge (z_1 \geqslant x_0 \Rightarrow MP_2 == MP_1 \wedge EQ_2 == EQ_1 \wedge CT_2 == CT_1 \wedge CS_2 == CS_1) \wedge (\neg(z_1 \geqslant x_0) \Rightarrow MP_2 == MP_0 \wedge EQ_2 == EQ_0 \wedge CT_2 == CT_0 \wedge CS_2 == CS_0) \wedge (\neg(z_1 \geqslant x_0) \Rightarrow m_2 == m_0) \wedge (\neg(z_1 \geqslant x_0) \Rightarrow n_2 == n_0))) \Rightarrow (true \Rightarrow MP_2 \geqslant 80000 \wedge EQ_2 \geqslant 60000 \wedge CT_2 \geqslant 10000 \wedge CS_2 \geqslant 5000)$

(b) Verification Condition generated based on f_{stmt} and f_{pr}

Figure 15: Verification Condition of DBprog^{dsa} based on CNF

tifiers in the formula, in this section, we extend VCG based on the weakest precondition to the case of database language. Given a program statement *stmt* and a postcondition ψ, the weakest precondition of iteration free imperative program is computed as follows:

$$\text{wp}(\texttt{skip}, \psi) = \psi \qquad \text{wp}(v := e, \psi) = \psi[e/v]$$
$$\text{wp}(stmt_1; stmt_2, \psi) = \text{wp}(stmt_1, \text{wp}(stmt_2, \psi))$$
$$\text{wp}(\texttt{if } b \texttt{ then } stmt_1 \texttt{ else } stmt_2 \texttt{ endif}, \psi) = (b \wedge \text{wp}(stmt_1, \psi)) \vee (\neg b \wedge \text{wp}(stmt_2, \psi))$$

In order to adopt VCG of database programs, we define wp on database statements in Figure 16. Observe that, this process does not require to convert input programs into DSA form.

Given a program $\mathcal{P}$ annotated with *assume* and *assert* in the form $\{assume\ \psi_1; \mathcal{P}; assert\ \psi_2;\}$, the VC is constructed as follows: $\text{wp}(assume\ \psi_1, \text{wp}(\mathcal{P}, \psi_2)) = \psi_1 \Rightarrow \text{wp}(\mathcal{P}, \psi_2)$. The complete algorithmic steps to generate the wp-based VC is depicted in Algorithm 3 and this is illustrated with our motivating example in Example 3.

Example 3. *Consider the motivating program* DBprog *depicted in Figure 1. The annotated form of* DBprog *is depicted in Figure 17.*
The verification condition using weakest precondition wp *of the above annotated program is computed as follows:*

Let $c \triangleq m := z - x;$

$$\text{wp}(\texttt{skip}, \psi) = \psi \qquad \text{wp}(v := e, \psi) = \psi[e/v]$$
$$\text{wp}(assume\ \psi_1, \psi_2) = \psi_1 \Rightarrow \psi_2 \qquad \text{wp}(assert\ \psi_1, \psi_2) = \psi_1 \wedge \psi_2$$
$$\text{wp}(\langle rs := \texttt{SELECT}(f(\vec{e}), r(\vec{h}(\vec{x})), \phi, g(\vec{e})), cond\rangle, \psi) = (\psi[\text{F}(\vec{a})/rs] \wedge cond) \vee (\psi \wedge \neg cond)$$
$$\text{wp}(\langle \vec{a} := \texttt{UPDATE}(\vec{e}), cond\rangle, \psi) = ((\psi \wedge \neg cond) \vee (\psi[\vec{e}/\vec{a}] \wedge cond))$$
$$\text{wp}(\langle \vec{a} := \texttt{INSERT}(\vec{e}), false\rangle, \psi) = \psi[\vec{e}/\vec{a}] \vee \psi$$
$$\text{wp}(\langle \vec{a} := \texttt{DELETE}(), cond\rangle, \psi) = \psi \wedge \neg cond$$
$$\text{wp}(stmt_1; stmt_2, \psi) = \text{wp}(stmt_1, \text{wp}(stmt_2, \psi))$$
$$\text{wp}(\texttt{if } b \texttt{ then } stmt \texttt{ endif}, \psi) = (b \wedge \text{wp}(stmt, \psi)) \vee (\neg b \wedge \psi)$$
$$\text{wp}(\texttt{if } b \texttt{ then } stmt_1 \texttt{ else } stmt_2 \texttt{ endif}, \psi) = (b \wedge \text{wp}(stmt_1, \psi)) \vee (\neg b \wedge \text{wp}(stmt_2, \psi))$$

Figure 16: wp computation on our database language

Algorithm 3: Weakest Precondition Computation

Input: Database program $\mathcal{P}$, specification ψ
Output: Single VC

1 Let $\mathcal{P}$ be a sequence of statements $s_1; s_2; \ldots; s_n;$
2 Annotate $\mathcal{P}$ in the form of $assume\ \psi_1; \mathcal{P}; assert\ \psi_2;$
3 Apply wp($\mathcal{P}$, ψ_2) which results into ψ_3
4 Apply wp($assume\ \psi_1$, ψ_3) which generates the VC as $\psi_1 \Rightarrow \psi_3$
5 Return VC
6 **End**

```
1.  CREATE OR REPLACE PROCEDURE DBprog (y int, x int) IS
2.  z int;
3.  m int;
4.  n int;
5.  assume MP ⩾ 80000 ∧ EQ ⩾ 60000 ∧ CT ⩾ 10000 ∧ CS ⩾ 5000;
6.  BEGIN
7.    SELECT TA INTO z FROM Budget WHERE Did = y;
8.    if (z ⩾ x) then
9.        m := z − x;
10.       n := m/4;
11.       UPDATE Budget SET MP = MP−n, EQ = EQ − n, CT = CT − n, CS
          = CS − n WHERE Did = y;
12.   endif
13. assert MP ⩾ 80000 ∧ EQ ⩾ 60000 ∧ CT ⩾ 10000 ∧ CS ⩾ 5000;
14. end;
```

Figure 17: Annotated form of DBprog

$n := m/4;$
UPDATE *Budget* SET MP = MP − n, EQ = EQ − n,
CT = CT − n, CS = CS − n WHERE Did = y;
and $\psi \triangleq MP \geqslant 80000 \wedge EQ \geqslant 60000 \wedge CT \geqslant 10000 \wedge CS \geqslant 5000$

Then $\text{wp}(c, \psi) \triangleq \text{wp}([$
$m := z - x;$
$n := m/4;$
UPDATE *Budget* SET MP = MP − n, EQ = EQ − n,
CT = CT − n, CS = CS − n WHERE Did = y;
$], \psi)$
$\triangleq \text{wp}([$
$m := z - x;$
$n := m/4;$

$$
\begin{aligned}
&\quad\ \ ], \psi_1)\\
&\quad\ \ \vdots\\
&= \psi_3\\
\text{Therefore, } &\mathsf{wp}([\\
&\quad \text{SELECT TA INTO } z \text{ FROM } \mathit{Budget} \text{ WHERE Did} = y;\\
&\quad \mathit{if}\ (z \geqslant x)\ \mathit{then}\\
&\quad\ \ c\\
&\quad \mathit{endif}\\
&\quad], \psi)\\
\triangleq\ &\mathsf{wp}([\\
&\quad \text{SELECT TA INTO } z \text{ FROM } \mathit{Budget} \text{ WHERE Did} = y;\\
&\quad], \psi_4)\\
&\text{where } \psi_4 = (z \geqslant x \wedge \psi_3) \vee (\neg(z \geqslant x) \wedge \psi)\\
&= \psi_5
\end{aligned}
$$

Finally, VC *is generated as follows:*

$$
\begin{aligned}
&\mathsf{wp}([\\
&\quad \mathit{assume}\ \text{MP} \geqslant 80000 \wedge \text{EQ} \geqslant 60000 \wedge \text{CT} \geqslant 10000 \wedge \text{CS} \geqslant 5000\\
&\quad], \psi_5)\\
&= \text{MP} \geqslant 80000 \wedge \text{EQ} \geqslant 60000 \wedge \text{CT} \geqslant 10000 \wedge \text{CS} \geqslant 5000 \implies \psi_5\\
&= (\text{MP} \geqslant 80000 \wedge \text{EQ} \geqslant 60000 \wedge \text{CT} \geqslant 10000 \wedge \text{CS} \geqslant 5000) \implies ((\text{Did}\\
&\quad = y \wedge \big(\text{TA} \geqslant x \wedge ((\text{Did} = y \wedge \text{MP} - (\text{TA} - x)/4 \geqslant 80000 \wedge \text{EQ} - (\text{TA}-\\
&\quad x)/4 \geqslant 60000 \wedge \text{CT} - (\text{TA} - x)/4 \geqslant 10000 \wedge \text{CS} - (\text{TA} - x)/4 \geqslant 5000)\\
&\quad \vee (\neg(\text{Did} = y) \wedge \text{MP} \geqslant 80000 \wedge \text{EQ} \geqslant 60000 \wedge \text{CT} \geqslant 10000 \wedge \text{CS} \geqslant\\
&\quad 5000))) \vee \big((\neg(\text{TA} \geqslant x) \wedge \text{MP} \geqslant 80000 \wedge \text{EQ} \geqslant 60000 \wedge \text{CT} \geqslant 10000 \wedge\\
&\quad \text{CS} \geqslant 5000))) \vee\ (\neg(\text{Did} = y) \wedge \big(z \geqslant x \wedge ((\text{Did} = y \wedge \text{MP} - (z - x)/4 \geqslant\\
&\quad 80000 \wedge \text{EQ} - (z - x)/4 \geqslant 60000 \wedge \text{CT} - (z - x)/4 \geqslant 10000 \wedge \text{CS}-\\
&\quad (z - x)/4 \geqslant 5000) \vee (\neg(\text{Did} = y) \wedge \text{MP} \geqslant 80000\ \wedge \text{EQ} \geqslant 60000 \wedge \text{CT} \geqslant\\
&\quad 10000 \wedge \text{CS} \geqslant 5000))) \vee ((\neg(z \geqslant x) \wedge \text{MP} \geqslant 80000 \wedge \text{EQ} \geqslant 60000 \wedge \text{CT}\\
&\quad \geqslant 10000 \wedge \text{CS} \geqslant 5000))))
\end{aligned}
$$

Note that, like SE *and* CNF *methods, in this case also the validity checking of* VC *yields unsatisfiablity for some cases, indicating property violation.*

Limitations: Although this approach generates a single verification condition, the length of the VC may be exponential w.r.t. program size. For instance, in the case of `if-else` statement, the weakest precondition computation considers both branches into a single formula, thus increasing its length. Therefore, a program with n conditional statements generates a single VC of length $O(2^n)$.

Efficient Weakest Precondition

We observed that for non-DSA programs, the weakest precondition technique produces VCs whose size is, in the worse case, exponential with respect to the size of the program. Flanagan and Saxe showed that when the technique was applied to DSA form of programs, the size of the generated VCs was, in the worst case, quadratic [43]. The main point to understanding the simplified definition of predicate transformers for DSA programs is to observe that the set of execution paths of such a program can be encoded logically in a compact way that does not require duplicating assert formula. We call this encoding the *program formula*. The program formula of an assignment statement is simply the corresponding equality, and the formula of a sequence of statements is the conjunction of formulas of the sub-statements. For conditional, the formula is $(b \wedge \psi_1) \vee (\neg b \wedge \psi_2)$, where ψ_1 and ψ_2 are the formulas of the branch statements. Therefore, the program formula of various statements of the language under consideration are depicted in Figure 18, where the function $\mathfrak{F}(\mathit{stmt}^{dsa})$ denotes the encoding of stmt^{dsa}.

$$
\begin{aligned}
&\mathfrak{F}(\mathtt{skip}) = \top \qquad \mathfrak{F}(\mathtt{assume}\ \phi^{dsa}) = \phi^{dsa} \qquad \mathfrak{F}(v_{i+1} := e_i) = \ v_{i+1} = e_i\\
&\mathfrak{F}(\langle rs_{i+1} := \mathtt{SELECT}(f(\vec{e}_i),\\
&\quad r(\vec{h}(\vec{a}_i)),\ \phi',\ g(\vec{e}_i)),\ cond_i\rangle) = ((cond_i \wedge rs_{i+1} = \mathtt{F}(\vec{a}_i))\\
&\qquad\qquad \vee (rs_{i+1} = rs_i))\\
&\mathfrak{F}(\langle \vec{a}_{i+1} := \mathsf{act}_i^1 \curvearrowright \mathsf{act}_i^2,\\
&\quad cond_i^1 \curvearrowright cond_i^2\rangle) = \ \mathfrak{F}(\langle \vec{a}_{i+1} := \mathsf{act}_i^1,\ cond_i^1\rangle)\\
&\qquad\qquad \vee \mathfrak{F}(\langle \vec{a}_{i+1} := \mathsf{act}_i^2,\ cond_i^2\rangle)\\
&\mathfrak{F}(\langle \vec{a}_{i+1} := \mathtt{UPDATE}(\vec{e}_i),\ cond_i\rangle) = (cond_i \wedge \bigwedge_{j=1}^{|\vec{a}_{i+1}|} a_{i+1}^j = e_i^j)\\
&\mathfrak{F}(\langle \vec{a}_{i+1} := \mathtt{INSERT}(\vec{e}_i),\ false\rangle) = \ (\bigwedge_{j=1}^{|\vec{a}_{i+1}|} a_{i+1}^j = e_i^j)\\
&\mathfrak{F}(\langle \vec{a}_{i+1} := \mathtt{DELETE}(),\ cond_i\rangle) = \top\\
&\mathfrak{F}(\mathit{stmt}_1^{dsa};\ \mathit{stmt}_2^{dsa}) = \mathfrak{F}(\ \mathit{stmt}_1^{dsa}) \ \wedge\ \mathfrak{F}(\mathit{stmt}_2^{dsa})\\
&\mathfrak{F}(\mathtt{if}\ b_i\ \mathtt{then}\ \mathit{stmt}_1^{dsa}\ \mathtt{else}\ \mathit{stmt}_2^{dsa}) = (b_i\ \wedge\ \mathfrak{F}(\mathit{stmt}_1^{dsa})\\
&\qquad\qquad \vee (\neg b_i\ \wedge \mathfrak{F}(\mathit{stmt}_2^{dsa}))\\
&\mathfrak{F}(\mathtt{assert}\ \phi^{dsa}) = \phi^{dsa}
\end{aligned}
$$

Figure 18: Definition of function $\mathfrak{F}(.)$ for language statements

Authors in [44] later proposed a simplified solution emphasizing that the technique could be seen as the special case of weakest precondition computation by introducing "dream property".

$$\mathsf{wlp}(\mathit{stmt}^{dsa},\ \psi) \triangleq \mathfrak{F}(\mathit{stmt}^{dsa}) \Rightarrow \psi$$

where $\mathsf{wlp}(\mathit{stmt}^{dsa},\ \psi)$ is the weakest formula (known as *weakest liberal precondition* of stmt^{dsa} w.r.t ψ), characterizes the pre-states from which all non-blocking execution of stmt^{dsa} either goes wrong or terminates in a state satisfying ψ.

Since WPC is conservative in the sense that $\mathsf{wp}(\mathcal{P}^{dsa},\ \psi)$ is a predicate over pre-states of $\mathcal{P}^{dsa}$ such that all possible executions of $\mathcal{P}^{dsa}$ terminates in a state satisfying ψ and in all execution the assertion ψ is satisfied. However, in $\mathsf{wlp}(\mathcal{P}^{dsa},\ \psi)$, the execution may go wrong or terminate in a state satisfying ψ. Therefore, the author in [44] derived a notable relation between wp and wlp as follows :

$$\mathsf{wp}(\mathcal{P}^{dsa},\ \psi) \ \triangleq\ \mathsf{wlp}(\mathcal{P}^{dsa},\ \psi) \ \wedge\ \mathsf{wp}(\mathcal{P}^{dsa},\ \mathit{true})$$

In case of no assert statements in $\mathcal{P}^{dsa}$, we have $\mathsf{wp}(\mathcal{P}^{dsa},\ \psi) = \mathsf{wlp}(\mathcal{P}^{dsa},\ \psi)$, since $\mathsf{wp}(\mathcal{P}^{dsa},\ \mathit{true}) = \mathit{true}$. Therefore, by following the dream property, wlp of a program $\mathcal{P}^{dsa}$ w.r.t. ψ can be computed as follows:

$$\mathsf{wlp}(\mathcal{P}^{dsa},\ \psi) \triangleq \mathfrak{F}(\mathcal{P}^{dsa}) \Rightarrow \psi$$

The wlp of programs is defined in the same way as of wp (as shown in Figure 16) except for *assume* and *assert* statements:

$$\mathsf{wlp}(assume\ \phi_1,\ \psi_1) = \phi_1 \Rightarrow \psi_1$$
$$\mathsf{wlp}(assert\ \phi_2,\ \psi_2) = \phi_2 \Rightarrow \psi_2$$

Therefore, for a given database program $\mathcal{P}^{dsa}$; *assert* ψ, where $\mathcal{P}^{dsa}$ does not contain other assert statements, the following function computes a single VC.

$$\mathsf{wp}(\mathcal{P}^{dsa},\ assert\ \psi,\ true) = \mathsf{wlp}(\mathcal{P}^{dsa},\ \psi) = \mathfrak{F}(\mathcal{P}^{dsa}) \Rightarrow \psi$$

Since $\mathfrak{F}(\mathcal{P}^{dsa})$ generates linear size program formula, unlike WPC-based VC, the size of efficient weakest precondition-based VC is linear in the size of the program. The following example illustrates this.

Example 4. *Consider annotated* DBprog^{dsa} *shown in Figure 11.*

$$\begin{aligned}
Let\ stmt_1^{dsa} =\ & assume\ \mathrm{MP}_0 \geqslant 80000\ and\ \mathrm{EQ}_0 \geqslant 60000\ and\ \mathrm{CT}_0 \geqslant 10000\\
& and\ \mathrm{CS}_0 \geqslant 5000;\\
& \text{SELECT } \mathrm{TA}_0 \text{ INTO } z_1 \text{ FROM Budget WHERE } \mathrm{Did}_0 = y_0;\\
stmt_2^{dsa} =\ & \text{if } z_1 \geqslant x_0 \text{ then}\\
& m_1 := z_1 - x_0;\\
& n_1 := m_1/4;\\
& \text{UPDATE Budget SET } \mathrm{MP}_1 = \mathrm{MP}_0 - n_1,\ \mathrm{EQ}_1 = \mathrm{EQ}_0 - n_1\\
& ,\mathrm{CT}_1 = \mathrm{CT}_0 - n_1,\ \mathrm{CS}_1 = \mathrm{CS} - n_1 \text{ WHERE } \mathrm{Did}_0 = x_0;\\
& n_2 := n_0;\\
& \text{endif;}\\
\psi =\ & assert\ \mathrm{MP}_2 \geqslant 80000\ and\ \mathrm{EQ}_2 \geqslant 60000\ and\ \mathrm{CT}_2 \geqslant 10000\\
& and\ \mathrm{CS}_2 \geqslant 5000;
\end{aligned}$$

$$\mathrm{wp}(\mathrm{DBprog}^{dsa}, \mathsf{assert}\ \psi,\ true) = \mathrm{wlp}(\mathrm{DBprog}^{dsa},\ \psi) = \mathfrak{F}(\mathrm{DBprog}^{dsa}) \Rightarrow \psi$$

$$\mathfrak{F}(\mathrm{DBprog}^{dsa}) \Rightarrow \psi = \mathfrak{F}(stmt_1^{dsa};\ stmt_2^{dsa}) \Rightarrow \psi$$

$$\mathfrak{F}(stmt_1^{dsa}\ ;\ stmt_2^{dsa}) = \mathfrak{F}(stmt_1^{dsa}) \wedge \mathfrak{F}(stmt_2^{dsa})$$

$$\mathfrak{F}(stmt_1^{dsa}) = ((\mathrm{MP}_0 \geqslant 80000 \wedge \mathrm{EQ}_0 \geqslant 60000 \wedge \mathrm{CT}_0 \geqslant 10000 \wedge \mathrm{CS}_0 \geqslant 5000) \wedge (z_1 = \mathrm{TA}_0 \wedge \mathrm{Did}_0 = y_0) \vee (z_1 = z_0 \wedge \neg(\mathrm{Did}_0 = y_0))$$

$$\mathfrak{F}(stmt_2^{dsa}) = ((z_1 \geqslant x_0 \wedge \mathfrak{F}(stmt_t^{dsa}) \vee (\neg z_1 \geqslant x_0 \wedge \mathfrak{F}(stmt_f^{dsa})))$$

where $\mathfrak{F}(stmt_t^{dsa}) = (m_1 = z_1 - x_0 \wedge n_1 = m_1/4 \wedge ((\mathrm{Did}_0 = x_0 \wedge \mathrm{MP}_1 = \mathrm{MP}_0 - n_1 \wedge \mathrm{EQ}_1 = \mathrm{EQ}_0 - n_1 \wedge \mathrm{CT}_1 = \mathrm{CT}_0 - n_1 \wedge \mathrm{CS}_1 = \mathrm{CS} - n_1) \vee (\neg(\mathrm{Did}_0 = x_0) \wedge \mathrm{MP}_1 = \mathrm{MP}_0 \wedge \mathrm{EQ}_1 = \mathrm{EQ}_0 \wedge \mathrm{CT}_1 = \mathrm{CT}_0 \wedge \mathrm{CS}_1 = \mathrm{CS}_0)) \wedge m_2 = m_1 \wedge n_2 = n_1 \wedge \mathrm{MP}_2 = \mathrm{MP}_1 \wedge \mathrm{EQ}_2 = \mathrm{EQ}_1 \wedge \mathrm{CT}_2 = \mathrm{CT}_1 \wedge \mathrm{CS}_2 = \mathrm{CS}_1)$

$$\mathfrak{F}(stmt_f^{dsa}) = (\mathrm{MP}_2 = \mathrm{MP}_0 \wedge \mathrm{EQ}_2 = \mathrm{EQ}_0 \wedge \mathrm{CT}_2 = \mathrm{CT}_0 \wedge \mathrm{CS}_2 = \mathrm{CS}_0 \wedge m_2 = m_0 \wedge n_2 = n_0)$$

$$\mathrm{wlp}(\mathrm{DBprog}^{dsa}, \psi) = ((\mathrm{MP}_0 \geqslant 80000 \wedge \mathrm{EQ}_0 \geqslant 60000 \wedge \mathrm{CT}_0 \geqslant 10000 \wedge \mathrm{CS}_0 \geqslant 5000) \wedge (z_1 = \mathrm{TA}_0 \wedge \mathrm{Did}_0 = y_0) \vee (z_1 = z_0 \wedge \neg(\mathrm{Did}_0 = y_0)) \wedge ((z_1 \geqslant x_0 \wedge (m_1 = z_1 - x_0 \wedge n_1 = m_1/4 \wedge \mathrm{Did}_0 = x_0 \wedge \mathrm{MP}_1 = \mathrm{MP}_0 - n_1 \wedge \mathrm{EQ}_1 = \mathrm{EQ}_0 - n_1 \wedge \mathrm{CT}_1 = \mathrm{CT}_0 - n_1 \wedge \mathrm{CS}_1 = \mathrm{CS} - n_1) \vee (\neg(\mathrm{Did}_0 = x_0) \wedge \mathrm{MP}_1 = \mathrm{MP}_0 \wedge \mathrm{EQ}_1 = \mathrm{EQ}_0 \wedge \mathrm{CT}_1 = \mathrm{CT}_0 \wedge \mathrm{CS}_1 = \mathrm{CS}_0) \wedge m_2 = m_1 \wedge n_2 = n_1 \wedge \mathrm{MP}_2 = \mathrm{MP}_1 \wedge \mathrm{EQ}_2 = \mathrm{EQ}_1 \wedge \mathrm{CT}_2 = \mathrm{CT}_1 \wedge \mathrm{CS}_2 = \mathrm{CS}_1) \vee ((\neg z_1 \geqslant x_0) \wedge \mathrm{MP}_2 = \mathrm{MP}_0 \wedge \mathrm{EQ}_2 = \mathrm{EQ}_0 \wedge \mathrm{CT}_2 = \mathrm{CT}_0 \wedge \mathrm{CS}_2 = \mathrm{CS}_0 \wedge m_2 = m_0 \wedge n_2 = n_0)) \Rightarrow \mathrm{MP}_2 \geqslant 80000 \wedge \mathrm{EQ}_2 \geqslant 60000 \wedge \mathrm{CT}_2 \geqslant 10000 \wedge \mathrm{CS}_2 \geqslant 5000$$

Observe that the above VC is linear in size and does not have duplicate assert expressions.

3.4. Addressing Aggregate Functions, NULL Values, Sub-query, JOIN, UNION, INTERSECT, and MINUS operations

This section provides guidance to deal with crucial database-specific features and operations.

Aggregate functions. The aggregate functions only appear in SELECT statements which usually return a single value as the answer to a posed query. This means that the violation is only possible if the values obtained through aggregate functions are stored in a result-set variable which in turn may affect, directly or indirectly, the specification representing database property.

Given a database program containing aggregate function $h(x)$ on attribute x, our VC-based approaches ensure the presence of $h(x)$ in the resultant VCs. If we treat $h(x)$ in the VCs as a new variable and if the VC violates the specification, let m be the value of $h(x)$ for which violation is observed. Since our main is to identify one instance which leads to properties violation, we adopt the following approach to deal with various aggregate functions:

(a) $h(x) \triangleq \mathsf{min}(x)$.
There exists a minimum values m of attribute x in a database instance which leads to this violation. However, this may be a false positive because of the treatment of x and $\mathsf{min}(x)$ as different variables. As a solution, we propose to adopt all the constraints over x as the constraints over $\mathsf{min}(x)$ in the VCs.

(b) $h(x) \triangleq \mathsf{max}(x)$.
Same as $\mathsf{min}(x)$.

(c) $h(x) \triangleq \mathsf{sum}(x)$.
In this case, we can adopt only the constraints which define the lowest bound of x-values as the constraints over the variable $\mathsf{sum}(x)$. Observe that this over-approximation may lead to false positive.

(d) $h(x) \triangleq \mathsf{avg}(x)$.
In this case, we can adopt only the constraints which define both the lower- and upper-bound of x-values as the constraints over the variable $\mathsf{avg}(x)$.

(e) $h(x) \triangleq \mathsf{count}(*)$.
This is applicable only when we consider table-level property as a part of the specification. An example of a table-level property is "*each department must have at least two employees*". As we consider only tuple-level property, this case is out of the scope of this work, and we consider it as our future plan.

Treating NULL values. In order to address NULL values, we provide a separate treatment to the properties which specify whether attribute values may accept NULL or must be NOT NULL. To be specific, NULL property-violation may take place in the following situations:

(a) The value of an expression e is set to an attribute "a" in INSERT or UPDATE, where "a" is NOT NULL and the value of e may be NULL.

(b) The presence of the condition in the form "x is NULL/x is NOT NULL" in conditional statement.

In case (a), special care should be taken by checking the possibility of NULL values occurs in e against the NULL/NOT NULL property of the attribute "a". In such a case, a warning report is generated. In case (b), the presence of condition in the form of "x IS NULL" or "x IS NOT NULL" always leads to " always false" or "always true" respectively w.r.t the given NULL constraints "x IS NOT NULL". Therefore, the VC generation can decide statically whether to ignore or to include the logical encoding of the other part of the database statement.

Sub-query. A sub-query can be nested inside the WHERE or HAVING clause of an outer SELECT, INSERT, UPDATE, or DELETE statement, or inside another sub-query. A sub-query can appear anywhere an expression can be used, if it returns a single value. SQL statements that include a sub-query take one of these following format:

… WHERE ⟨e⟩⟨*comparison_operator*[ANY | ALL]⟩⟨(sub-query)⟩.
… WHERE ⟨e [NOT] IN⟩⟨(sub-query)⟩.
… WHERE ⟨[NOT] EXIST⟩⟨(sub-query)⟩

In the case of sub-query of the format "… WHERE ⟨e⟩ ⟨*comparison_operator*[ANY | ALL]⟩ ⟨(sub-query)⟩", we first convert the inner query into a logical sub-formula which become a part of the final VC involving attributes, operators and co-relation present in the outer query. Example 5 illustrates this scenario.

Example 5. *Consider the* UPDATE *statement* $Q_{upd} \triangleq$ UPDATE *ROUTES* SET P = P $*$ ((100 − d)/100) WHERE R_ID ⩾ (SELECT L_ID FROM *LOADS* WHERE CID = z);. *The logical sub-formula generated from the sub-query using symbolic execution is* $\mathrm{R\_ID}_0 \geqslant \mathrm{L\_ID}_0 \wedge \mathrm{CID}_0 = z_0$. *Therefore, the resultant VC would be:*

$$((\mathrm{P}_1 = \mathrm{P}_0 * ((100 - d_0)/100) \wedge \mathrm{R\_ID}_0 \geqslant \mathrm{L\_ID}_0 \wedge \mathrm{CID}_0 = z_0) \vee (\mathrm{P}_1 = \mathrm{P}_0 \wedge \neg(\mathrm{R\_ID}_0 \geqslant \mathrm{L\_ID}_0 \wedge \mathrm{CID}_0 = z_0)))$$

In the case of nested query of the format "… WHERE ⟨e [NOT] IN⟩ ⟨(sub-query)⟩", we replace the IN operator by assignment operator (:=) during VC generation. Since our main aim is to determine property violation by the database code, any instance invalidating the resultant VC fulfills our objective.

JOIN. Without loss of generality, let us assume that all attributes names in the database are unique. The presence of a query containing θ-JOIN is addressed easily by incorporating the condition θ in the VC, in addition. Observe that equi-JOIN and natural-JOIN are special cases of θ-JOIN.

UNION, INTERSECT and MINUS operations. Two or more queries can be combined using set operators UNION, INTERSECT and MINUS. The logical encoding of UNION (or INTERSECT) is achieved by using logical OR ∨ (or logical AND ∧) operator. That is, logical formulas obtained from operands of UNION (or INTERSECT) are composed using ∨ (or ∧). Since MINUS operation can be replaced by UNION and INTERSECT operations, VC generation in the presence of MINUS can be achieved by using ∨ and ∧.

3.5. Treating Loops

As mentioned earlier, deductive verification approaches require user's guidance and expertise for program annotation. Naturally, the presence of loops in a program makes this process more challenging. The fundamental step in such a case is to infer a loop invariant which remains true throughout the loop iterations.

Following are the situations where iteration is required for effective coding for database programs:

- If the action over the tuples of a table is parameterized with changeable parameters for different tuples. In this case, rather than writing separate database statements in the code for different tuples of the table, the parameterized action can collectively be expressed in terms of a loop.
- To iterate over the result-set values using a cursor.
- Recursive Queries.

The relation between Hoare triples and weakest precondition is that $\{\phi\}\mathcal{P}\{\psi\}$ iff $\phi \implies \mathsf{wp}(\mathcal{P}, \psi)$ or $\mathsf{sp}(\mathcal{P}, \phi) \implies \psi$. Hoare logic was proposed to deal with iterating While-programs, based on the loop invariant. Formally, the following classic inference rule [35] uses the invariant ϕ to express the partial correctness of any loop:

$$\frac{\vdash \{\phi \wedge cond\}stmt\{\phi\}}{\vdash \{\phi\}\text{while } cond \text{ do } stmt \{\phi \wedge \neg cond\}}$$

Figure 19 defines the Hoare logic H_{db} for the database programs by extending the same for imperative programs. Observe that, since wp generates quantifier free formulas, this is the reason why wp is often used in Hoare logic verifier, instead of sp. While the computation of loop invariants in

$$\{\psi\} \text{ skip } \{\psi\} \quad \text{(Skip)}$$

$$\{\psi[e/v_a]\}\ v_a := e\ \{\psi\} \quad \text{(Assignment)}$$

$$\frac{\{(\psi[\vec{e}/\vec{a}] \wedge cond) \vee (\psi \wedge \neg cond)\}\langle \vec{a} := \mathtt{UPDATE}(\vec{e}),\ cond\rangle\{\psi\}}{} \quad \text{(UPDATE)}$$

$$\{\psi[\vec{e}/\vec{a}] \vee \psi\}\langle \vec{a} := \mathtt{INSERT}(\vec{e}),\ false\rangle\{\psi\} \quad \text{(INSERT)}$$

$$\{\psi \wedge \neg cond\}\langle \vec{a} := \mathtt{DELETE}(),\ cond\rangle\{\psi\} \quad \text{(DELETE)}$$

$$\frac{\{(\psi[\mathrm{F}(\vec{a})/rs] \wedge cond)) \vee (\psi \wedge \neg cond)\}}{\langle rs := \mathtt{SELECT}(f(\vec{e}), r(\vec{h}(\vec{x})), \phi, g(\vec{e})),\ cond\rangle\{\psi\}} \quad \text{(SELECT)}$$

$$\frac{\{\phi\}stmt_1\{\phi'\} \qquad \{\phi'\}stmt_2\{\psi\}}{\{\phi\}stmt_1; stmt_2\{\psi\}} \quad \text{(Sequence)}$$

$$\frac{\{\phi \wedge b\}stmt_1\{\psi\} \qquad \{\phi \wedge \neg b\}stmt_2\{\psi\}}{\{\phi\}\text{if } b \text{ then } stmt_1 \text{ else } stmt_2\{\psi\}} \quad \text{(Conditional)}$$

$$\frac{\{\phi \wedge b\}stmt\{\phi\}}{\{\phi\}\text{while } b \text{ do } stmt\{\phi \wedge b\}} \quad \text{(While loop)}$$

Figure 19: H_{db}: Hoare rules for database programs

host imperative languages relies on existing approaches [35],

the same can be adopted for the database as well. In particular, the computation of inductive invariants in these cases, by following the approaches in [45, 46], can be used with our proposed verification approaches. On the other hand, as an alternative solution, we can also extend the existing works on an abstract interpretation of database programs [34]. The definition of widening operation, in addition, covers recursive queries as well [47].

Table 3
Database before and after the update operation

(a) table t

eid	*sal*	*age*	*dno*
1	1500	35	10
2	800	28	20
3	2500	50	10
4	3000	62	10

(b) table t''

eid	*sal*	*age*	*dno*
1	1600	35	10
2	800	28	20
3	2600	50	10
4	3100	62	10

4. Complexity Analysis and Correctness Proofs

We are now in a position to perform complexity analysis of the proposed VC generation algorithms and to prove their correctness.

4.1. Complexity Analysis

Let us describe the asymptotic characteristics of VCs generation by various approaches. VC generation based on symbolic execution generates VCs along all execution paths of a program. For a given program with n conditional statements, there exist 2^n execution paths, and therefore this results in $O(2^n)$ number of VCs. In contrary, the CNF-based approach sequentializes branches of conditional statements which increases the program's length to some extent. Assuming that the program contains a chain of nested conditions up to depth m, the approach generates a single VC of size $O(n + m^2)$. The weakest precondition-based approach considers both branches into a single formula. Therefore, a program with n conditional statements generates a single VC of size $O(2^n)$.

4.2. Correctness Proofs

We define a number of functions, namely Path, VC^{se}, VC^{cnf} and wp, which act as a core of different VC generation techniques in Section 3. Therefore, the correctness of the proposed verification techniques can be guaranteed by proving the correctness of these functions. We achieve this by considering the validity of input and output logical formulas in terms of their semantics w.r.t. states and transition semantics of database applications.

To this aim, let us first recall from [34] the notions of states and state transition semantics of database programs.

State Transition Semantics of Database Language. Since our database language involves both host imperative variables and database attributes, the state is defined by considering the semantics domains for both of them.

Definition 1 (Application Environment)**.** *Given the set of application variables $\mathbb{V}_a$ and the domain of values* `Val`*, let $\mathfrak{E}_a \triangleq [\mathbb{V}_a \mapsto \texttt{Val}]$ be the set of all functions with domain $\mathbb{V}_a$ and range included in* `Val`*. An application environment $\rho_a \in \mathfrak{E}_a$ maps application variables to the domain of values* `Val`*.*

Definition 2 (Database Environment)**.** *A database d is a set of tables $\{t_i \mid i \in I_x\}$ for a given set of indexes I_x. A database environment is defined as a function ρ_d whose domain is I_x, such that for $i \in I_x$, $\rho_d(i) = t_i$.*

Definition 3 (Table Environment)**.** *Given a database table t with attribute* `attr(t)`*$=\{a_1, a_2, \ldots, a_k\}$. So, $t \subseteq D_1 \times D_2 \times .... \times D_k$ where a_i is the attribute corresponding to the typed domain D_i. A* table environment *ρ_t for a table t is defined as a function such that for any attribute $a_i \in$* `attr(t)`*, $\rho_t(a_i) = \langle \pi_i(l_j) \mid l_j \in t \rangle$ where π is the projection operator and $\pi_i(l_j)$ represents i^{th} element of the l_j-th row. In other words, ρ_t maps a_i to the ordered set of values over the rows of the table t.*

Definition 4 (States and Concrete Semantics)**.** *Let Σ_{dba} be the set of states for the database language under consideration, defined by $\Sigma_{dba} \triangleq \mathfrak{E}_{dbs} \times \mathfrak{E}_{aps}$ where $\mathfrak{E}_{dbs}$ and $\mathfrak{E}_{aps}$ denote the set of all database environments and the set of all application environments respectively. Therefore, a state $\rho \in \Sigma_{dba}$ is denoted by a tuple (ρ_d, ρ_a) where $\rho_d \in \mathfrak{E}_{dbs}$ and $\rho_a \in \mathfrak{E}_{aps}$. The transition relation*

$$\mathcal{T}_{dba} \in \left[(\mathbb{C} \times \Sigma_{dba}) \mapsto \wp(\Sigma_{dba})\right] \tag{2}$$

specifies which successor states $(\rho_{d'}, \rho_{a'}) \in \Sigma_{dba}$ can follow when a statement $c \in \mathbb{C}$ executes on state $(\rho_d, \rho_a) \in \Sigma_{dba}$. Therefore, the transitional semantics $\mathcal{T}_{dba}[\![\mathcal{P}]\!] \in \left[(\mathcal{P} \times \Sigma_{dba}[\![\mathcal{P}]\!]) \mapsto \wp(\Sigma_{dba}[\![\mathcal{P}]\!])\right]$ of a program $\mathcal{P}$ restricts the transition relation to program instructions only, i.e.

$$\mathcal{T}_{dba}[\![\mathcal{P}]\!](\rho_d, \rho_a) = \{(\rho_{d'}, \rho_{a'}) \mid (\rho_d, \rho_a), (\rho_{d'}, \rho_{a'}) \in \Sigma_{dba}[\![\mathcal{P}]\!] \wedge c \in \mathcal{P} \wedge (\rho_{d'}, \rho_{a'}) \in \mathcal{T}_{dba}[\![c]\!](\rho_d, \rho_a)\}$$

Example 6 illustrates the concrete semantics of an update statement.

Example 6. *Consider the database table t in Table 3(a) and the following update statement:*

$$Q_{upd} : \texttt{UPDATE}\ t\ \texttt{SET}\ sal := sal + 100\ \texttt{WHERE}\ age \geqslant 35$$

The abstract syntax is denoted by $\langle \texttt{UPDATE}(\vec{v_d}, \vec{e}),\ \phi \rangle$, where $\phi = (age \geqslant 35)$ and $\vec{v_d} = \langle sal \rangle$ and $\vec{e} = \langle sal + 100 \rangle$.

The table targeted by Q_{upd} is $target(Q_{upd})$= {t}. The semantics of Q_{upd} is:

$$\begin{aligned}
&\mathcal{T}_{dba}[\![Q_{upd}]\!](\rho_d, \rho_a) \\
&= \mathcal{T}_{dba}[\![\left\langle \texttt{UPDATE}(\langle sal \rangle, \langle sal + 100 \rangle),\ (age \geqslant 35) \right\rangle]\!](\rho_d, \rho_a) \\
&= \mathcal{T}_{dba}[\![\left\langle \texttt{UPDATE}(\langle sal \rangle, \langle sal + 100 \rangle),\ (age \geqslant 35) \right\rangle]\!](\rho_t, \rho_a)
\end{aligned}$$

$$
\begin{aligned}
&[\textbf{\textit{Since,}}\ target(Q_{upd})=\{t\}]\\
=&\mathcal{T}_{dba}[\![\texttt{UPDATE}(\langle sal\rangle, \langle sal+100\rangle)]\!](\rho_{t\downarrow(age\geqslant 35)}, \rho_a) \sqcup (\rho_{t\downarrow\neg(age\geqslant 35)}, \rho_a)\\
&[\textbf{\textit{Absorbing}}\ \phi = (age \geqslant 35)]\\
=&(\rho_{t'}, \rho_a) \sqcup (\rho_{t\downarrow\neg(age\geqslant 35)}, \rho_a) = (\rho_{t'} \sqcup \rho_{t\downarrow\neg(age\geqslant 35)},\ \rho_a \sqcup \rho_a) = (\rho_{t''}, \rho_a)\\
&where\ \rho_{t'} \equiv \rho_{t\downarrow(age\geqslant 35)}\left[sal \leftarrow E[\![sal+100]\!](\rho_{t\downarrow(age\geqslant 35)}, \rho_a)\right]\\
&\qquad = \rho_{t\downarrow(age\geqslant 35)}\left[sal \leftarrow \langle 1600, 2600, 3100\rangle\right]
\end{aligned}
$$

The notation $(t \downarrow (age \geqslant 35))$ denotes the set of tuples in t for which $(age \geqslant 35)$ is true (denoted by red part in t of Table 3(a)). $E[\![.]\!]$ is a semantic function for arithmetic expressions which maps "$sal + 100$" to a list of values $\langle 1600, 2600, 3100\rangle$ on the table environment $\rho_{t\downarrow(age\geqslant 35)}$. The notation $\leftarrow$ denotes a substitution by new values. Observe that the substitution of 'sal' by the list of values in $\rho_{t\downarrow(age\geqslant 35)}$ results into a new table environment $\rho_{t'}$ (denoted by red part in Table 3(b)). Finally, the least upper bound (denoted $\sqcup$) of the two states results into a new state $(\rho_{t''}, \rho_a)$ where t'' is depicted in Table 3(b).

Lemmas and Theorems. Let us now state a number of lemmas and theorems aiming to guarantee the correctness of various VC generation functions in our verification approaches. As mentioned earlier, we make use of state and transition semantics defined above to prove these lemmas and theorems.

Extending the semantics of assertion language defined in [35] to the case of database applications, let the relation $(\rho_d, \rho_a) \vDash_I \Phi$ means that the state (ρ_d, ρ_a) satisfies the assertion Φ under the interpretation I.

Lemma 1. *Let Φ be the logical encoding of an execution path π up to program point ℓ. Let $stmt^{dsa}$ be a DSA form of program statement at program point $\ell+1$. Let Ψ be the logical encoding computed using the function* Path$(\Phi, stmt^{dsa})$. *The function* Path *is correct if $\forall\ (\rho_d, \rho_a) :\ (\rho_d, \rho_a) \vDash_I \Phi$ under interpretation I and $\mathcal{T}_{dba}[\![stmt^{dsa}]\!](\rho_d, \rho_a) = (\rho_{d'}, \rho_{a'})$, implies $(\rho_{d'}, \rho_{a'}) \vDash_I \Psi$.*

Proof. The proof is established based on structural induction. Assume that (ρ_d, ρ_a) satisfies Φ under the interpretation I, i.e. $(\rho_d, \rho_a) \vDash_I \Phi$.

Assignment Statement $v_{i+1} := e_i$. Let $\mathcal{T}_{dba}[\![v_{i+1} := e_i]\!](\rho_d, \rho_a) = (\rho_{d'}, \rho_{a'})$, where $(\rho_{d'}, \rho_{a'})$ is obtained by substituting all occurrences of v_{i+1} in (ρ_d, ρ_a) by $\mathcal{T}_{dba}[\![e_i]\!](\rho_d, \rho_a)$. Since e_i does not involve v_{i+1}, its semantics are same w.r.t. both the states (ρ_d, ρ_a) and $(\rho_{d'}, \rho_{a'})$. Therefore,

$$(\rho_{d'}, \rho_{a'}) \vDash_I (v_{i+1} == e_i) \tag{3}$$

On the other hand, since v_{i+1} is not present in Φ due to DSA property and only v_{i+1} is affected in $(\rho_{d'}, \rho_{a'})$, we have

$$(\rho_{d'}, \rho_{a'}) \vDash_I \Phi \tag{4}$$

Combining Equations 3 and 4, we get $(\rho_{d'}, \rho_{a'}) \vDash_I (\Phi \wedge v_{i+1} == e_i)$.

Database Statements Q. Since a database statement Q involves action 'A' and condition 'cond', this can be treated as a guarded command equivalent to "if *cond* then A". Consider the DSA form $\langle \vec{a}_{i+1} := \mathsf{act}_i^1 \curvearrowright \mathsf{act}_i^2, cond_i^1 \curvearrowright cond_i^2\rangle$ of different database statements. By the definition of the function Path, we have:

$$
\begin{aligned}
\mathsf{Path}(\Phi,\ \langle \vec{a}_{i+1} :=\ &\mathsf{act}_i^1 \curvearrowright \mathsf{act}_i^2,\ cond_i^1 \curvearrowright cond_i^2\rangle) =\\
&\mathsf{Path}(\Phi, \langle \vec{a}_{i+1} := \mathsf{act}_i^1,\ cond_i^1\rangle) \vee\\
&\mathsf{Path}(\Phi, \langle \vec{a}_{i+1} := \mathsf{act}_i^2,\ cond_i^2\rangle)
\end{aligned}
$$

Let us now prove the lemma for each database action:

- $Q_{upd}^{dsa} \triangleq \langle \vec{a}_{i+1} := \texttt{UPDATE}(\vec{a}_i) \curvearrowright \texttt{UPDATE}(\vec{e}_i), \neg cond_i \curvearrowright cond_i\rangle$. Given a state $(\rho_d,\ \rho_a)$, assume that $\mathcal{T}_{dba}[\![Q_{upd}^{dsa}]\!]\ (\rho_d, \rho_a) = (\rho_{d'}, \rho_{a'})$. By the definition of the function Path, we have

$$
\begin{aligned}
\Psi = \mathsf{Path}(\Phi,\ Q_{upd}^{dsa}) = \Phi \wedge ((\neg cond_i \wedge \bigwedge_{j=1}^{|\vec{a}_{i+1}|} a_{i+1}^j == a_i^j) \vee\\
(cond_i \wedge \bigwedge_{j=1}^{|\vec{a}_{i+1}|} a_{i+1}^j == e_i^j))
\end{aligned}
$$

  Since $(\rho_d, \rho_a) \vDash_I \Phi$ and Φ does not involve $\vec{a}_{i+1}$ due to DSA property, the following holds.

$$(\rho_{d'}, \rho_{a'}) \vDash_I \Phi \tag{5}$$

  Now we have two possibilities: either $(\rho_{d'}, \rho_{a'}) \vDash_I \neg cond_i$ or $(\rho_{d'}, \rho_{a'}) \vDash_I cond_i$. In the former case, the action $\vec{a}_{i+1} := \texttt{UPDATE}(\vec{a}_i)$ results into the same semantics for $\vec{a}_i$ and $\vec{a}_{i+1}$ w.r.t. $(\rho_{d'}, \rho_{a'})$, i.e. $\mathcal{T}_{dba}[\![\vec{a}_i]\!](\rho_{d'}, \rho_{a'}) = \mathcal{T}_{dba}[\![\vec{a}_{i+1}]\!](\rho_{d'}, \rho_{a'})$. Therefore,

$$(\rho_{d'}, \rho_{a'}) \vDash_I (\neg cond_i \wedge \bigwedge_{j=1}^{|\vec{a}_{i+1}|} a_{i+1}^j == a_i^j) \tag{6}$$

  In the latter case, the attributes $a_{i+1}^j \in \vec{a}_{i+1}$ in $(\rho_{d'}, \rho_{a'})$, where $j = 1 \ldots |\vec{a}_{i+1}|$, are substituted by $\mathcal{T}_{dba}[\![e_i^j]\!]\ (\rho_d, \rho_a)$ respectively. Since only the value of $\vec{a}_{i+1}$ is affected in $(\rho_{d'}, \rho_{a'})$, we have $\mathcal{T}_{dba}[\![\vec{a}_{i+1}]\!](\rho_{d'},\ \rho_{a'}) = \mathcal{T}_{dba}[\![\vec{e}_i]\!](\rho_{d'},\ \rho_{a'})$, and therefore,

$$(\rho_{d'}, \rho_{a'}) \vDash_I (cond_i \wedge \bigwedge_{j=1}^{|\vec{a}_{i+1}|} a_{i+1}^j == e_i^j) \tag{7}$$

  Hence, combining Equations 5, 6 and 7, $(\rho_{d'},\ \rho_{a'}) \vDash_I \Psi$ is proved.

Proofs for other statements follow similar direction. Please see Appendix A.1 for details. □

Theorem 1. *Given a database program $\mathcal{P}$ and its annotated DSA form $\{assume\ \phi_1^{dsa}; \mathcal{P}^{dsa}; assert\ \phi_2^{dsa};\}$, let X be the set of verification conditions derived by the function VC^{se} which includes initial condition ϕ_1^{dsa}. If for all $\omega_i \in X$, ω_i is valid (denoted by $\vdash \omega_i$), then $\mathcal{P}$ satisfies the property ϕ_2^{dsa} and vice versa.*

Proof. On applying $\mathsf{VC}^{se}(\emptyset,\ \{assume\ \phi_1^{dsa}; \mathcal{P}^{dsa}; assert\ \phi_2^{dsa}\ ;\})$, according to the definition in Figure 10, we get a set of VCs X along all execution paths, each of the form $\omega_i \triangleq (\Phi_i \implies \phi_2^{dsa}) \in X$. For all initial states $(\rho_d,\ \rho_a)$ satisfying ϕ_1^{dsa}, i.e. $(\rho_d,\ \rho_a) \vDash_I \phi_1^{dsa}$, if $\mathcal{T}_{dba}[\![\{assume\ \phi_1^{dsa}; \mathcal{P}^{dsa}\}]\!](\rho_d,\ \rho_a) = (\rho_{d'},\ \rho_{a'})$, then according to Lemma 1 we have

$$for\ all\ \omega_i :\ \ (\rho_{d'},\ \rho_{a'}) \vDash_I \Phi_i \tag{8}$$

If the program satisfies the assert ϕ_2^{dsa}, this means $(\rho_{d'}, \rho_{a'}) \vDash_I \phi_2^{dsa}$. Therefore, all $\omega_i \triangleq (\Phi_i \implies \phi_2^{dsa})$ are valid. This proves the theorem. □

Theorem 2. *Let ω be a verification condition generated from annotated database program $\{assume\ \phi_1^{dsa};\ \mathcal{P}^{cnf};\ assert\ \phi_2^{dsa};\}$ in its CNF form, by applying the function VC^{cnf} which includes initial condition ϕ_1^{dsa}. If ω is valid (denoted as $\vdash\ \omega$), then $\mathcal{P}$ satisfies the property ϕ_2^{dsa} and vice versa.*

Proof. Since $\mathcal{P}^{cnf}$ is semantically equivalent to $\mathcal{P}^{dsa}$, the proof is similar to Theorem 1. □

Theorem 3. *Given an annotated program $\{assume\ \phi_1;\ \mathcal{P};\ assert\ \phi_2;\}$. If the verification condition $\phi_1 \implies wp(\mathcal{P}, \phi_2)$ is valid, then $\mathcal{P}$ satisfies the property ϕ_2 and vice versa.*

Proof. The proof is based on structural induction on $stmt \in \mathcal{P}$. Recall the definition of wp in Figure 16 and let $\mathsf{wp}(stmt, \psi_2) = \psi_1$. We have to prove that, if $(\rho_d, \rho_a) \vDash_I \phi_1$ and $\phi_1 \rightarrow \psi_1$ and $\psi_2 \rightarrow \phi_2$ and $\mathcal{T}_{dba}[\![stmt]\!](\rho_d, \rho_a) = (\rho_{d'}, \rho_{a'})$, then $(\rho_{d'}, \rho_{a'}) \vDash_I \psi_2$. Let us now consider the different cases:

Assignment $\triangleq v := e$. Let $(\rho_d,\ \rho_a) \vDash_I \phi_1$. According to the definition, $\mathsf{wp}(v := e,\ \psi_2) = \psi_2[e/v]$. Assume that $\phi_1 \rightarrow \psi_2[e/v]$. Therefore,

$$(\rho_d,\ \rho_a) \vDash_I \psi_2[e/v] \tag{9}$$

According to the transition semantics, assume $\mathcal{T}_{dba}[\![v := e]\!](\rho_d, \rho_a) = (\rho_{d'},\ \rho_{a'})$ such that

$$\rho_a(z) = \begin{cases} m,\ if\ z = v \\ \rho_a(z),\ Otherwise \end{cases}$$

where $m = \mathcal{T}_{dba}[\![e]\!](\rho_d, \rho_a)$. Since, in $(\rho_{d'},\ \rho_{a'})$ all occurrences of x are replaced by m, according to Equation 9, we have $(\rho_{d'},\ \rho_{a'}) \vDash_I \Psi_2$. Assuming $\psi_2 \rightarrow \phi_2$, this case is proved.

$Q_{upd} \triangleq \langle \vec{a} := \mathsf{UPDATE}(\vec{e}), cond \rangle$. Let $(\rho_d, \rho_a) \vDash_I \phi_1$. According to the definition, $\mathsf{wp}(\langle \vec{a} := \mathsf{UPDATE}(\vec{e}), cond\rangle,\ \psi_2) = ((\psi_2 \wedge \neg cond) \vee (\psi_2[\vec{e}/\vec{a}] \wedge cond))$. Assume that $\phi_1 \rightarrow ((\psi_2 \wedge \neg cond) \vee (\psi_2[\vec{e}/\vec{a}] \wedge cond))$. Therefore,

$$(\rho_d,\ \rho_a) \vDash_I ((\psi_2 \wedge \neg cond) \vee (\psi_2[\vec{e}/\vec{a}] \wedge cond)) \tag{10}$$

According to the semantics function, let us assume $\mathcal{T}_{dba}[\![Q_{upd}]\!](\rho_d, \rho_a) = (\rho_{d'}, \rho_{a'})$, where

(i) Values of $\vec{a}$ will be updated by $\mathcal{T}_{dba}[\![\vec{e}]\!](\rho_d, \rho_a)$ for the tuples satisfying '*cond*'. That is, like assignment statement, we have

$$(\rho_{d'}, \rho_{a'}) \vDash_I \Psi_2\ when\ (\rho_d, \rho_a) \vDash_I \Psi_2[\vec{e}/\vec{a}] \wedge cond \tag{11}$$

(ii) Values of $\vec{a}$ will remain unchanged for those tuples which do not satisfy '*cond*'. That is,

$$(\rho_{d'},\ \rho_{a'}) \vDash_I \Psi_2\ when\ (\rho_d,\ \rho_a) \vDash_I \Psi_2 \wedge \neg cond \tag{12}$$

Combining Equations 10, 11 and 12, the lemma is proved for Q_{upd} assuming $\Psi_2 \rightarrow \phi_2$.

Proofs for other statements follow similar direction. Please see Appendix A.1 for details. □

Recall the Hoare triple in Section 3.3 and the Hoare logic system H_{db} for database language defined in defined in Figure 19. The semantics of Hoare triple is defined below:

Definition 5. *The Hoare triple $\{\phi\}stmt\{\psi\}$ is said to be valid, denoted as $\vDash \{\phi\}stmt\{\psi\}$, whenever for all $((\rho_d,\ \rho_a),\ (\rho_{d'},\ \rho_{a'})) \in \Sigma$, if $(\rho_d,\ \rho_a) \vDash_I \phi$ and $\mathcal{T}_{dba}[\![stmt]\!](\rho_d,\ \rho_a) = (\rho_{d'},\ \rho_{a'})$, then $(\rho_{d'},\ \rho_{a'}) \vDash_I \psi$.*

Let us now prove the soundness of H_{db}. We denote by $\vdash_{H_{db}} \{\phi\}\mathcal{P}\{\psi\}$ the fact that Hoare triple $\{\phi\}\mathcal{P}\{\psi\}$ derivable by Hoare logic system H_{db}.

Theorem 4 (**Soundness of Hoare logic system.**). *Let $stmt \in \mathcal{P}$ and let ϕ, ψ be the pre-condition and post-condition respectively. If $\vdash_{H_{db}} \{\phi\}stmt\{\psi\}$, then $\vDash \{\phi\}stmt\{\psi\}$.*

Proof. The proofs for database statements follow the proof in Theorem 3. For assignment, sequence, conditional, and iteration statements, the reader may refer [35]. □

5. DBverify: A Database Code Verifier

In this section, we present DBverify, a prototype implementation of our proposed verification approaches for PL/SQL language, which is developed in Python with roughly 6,500 lines of codes. The workflow of DBverify is shown in Figure 20. DBverify consists of four modules: (1) `DSA-translator`, (2) `SE-verify`, (3) `CNF-verify`, and (4) `WP-verify`. The overall schematic diagrams of DBverify is depicted in Figure 21. Observe that the first module converts a given PL/SQL code into its equivalent DSA form. The modules `SE-verify`, `CNF-verify`, and `WP-verify` implement verification condition generation under three deductive-based approaches. Let us now describe each of the modules in detail.

(1) Module `DSA-translator`: This module converts a given PL/SQL code into its equivalent DSA form. The initial tasks of the module are to annotate PL/SQL code by *assume* and *assert* statements taking the given

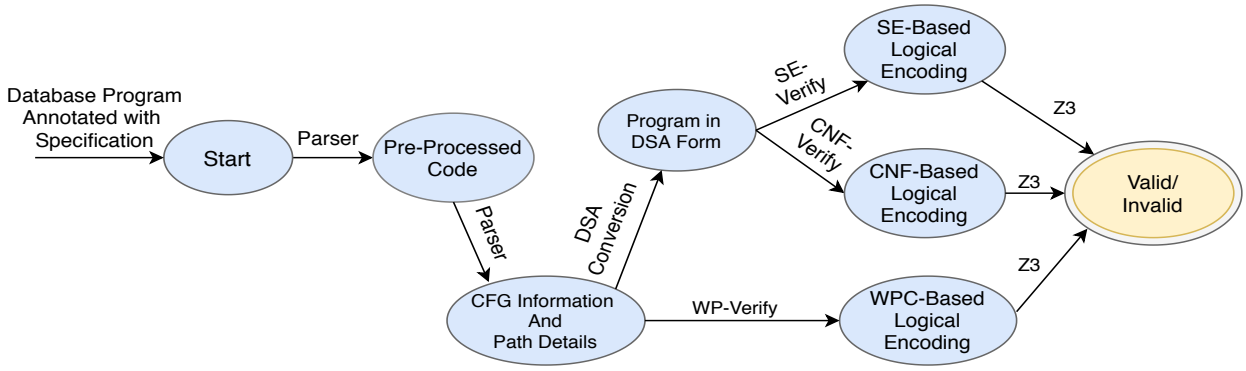


Figure 20: DBverify Workflow

specification into consideration, and then to construct the Control Flow Graph (CFG) of the annotated code using parsing techniques. We have used the ANTLR4 parser [31] for this purpose. Finally, taking CFG information as input, the module performs variable versioning and destruction of ϕ-nodes into its immediate predecessor-blocks to generate DSA form by following the standard algorithm [37] with an extension to cover the PL/SQL language. The schematic diagram of this module is depicted in Figure 21(a).

(2) Module `SE-verify`: Given an annotated PL/SQL program in its DSA form, this module generates a set of VCs according to Algorithm 1 depicted in section 3.1. The resultant VCs are further passed to the SMT Solver for validity checking. We have used Z3 for this purpose. The schematic diagram is depicted in Figure 21(b).

(3) Module `CNF-verify`: Unlike `SE-verifier`, this module first converts the annotated input program (in DSA form) into its equivalent CNF representation and then generates a single VC according to Algorithm 2 depicted in Section 3.2. Finally, this VC is fed to the Z3 SMT Solver for its validity checking. The schematic diagram of this module is depicted in Figure 21(c).

(4) Module `WP-verify`: For a given database program and its specification, this module computes a single VC according to Algorithm 3 in Section 3.3. The schematic diagram of this module is depicted in Figure 21(d).

As suggested in Section 3.4, DBverify considers the presence of aggregate operations as new variables. To deal with NULL, we maintain a dictionary of the form {tableName : [(attr-1, nullity), (attr-2, nullity), ..., (attr-n, nullity)]}, where nullity represents attribute constraint such as NULL or NOT NULL. On assigning an expression containing NULL to any of these attributes, we check and report (if any) constraint violations. When a NULLvalue appears in the guard of any conditional statements (e.g. `if` ... IS NULL {...}), we replace it by $*$ representing non-determinism. Therefore, the VC generation can decide statically whether to ignore or to include the logical encoding of the other part of the database statement. In the case of nested queries, DBverify processes the query from the inner-most sub-query to the outer-most sub-query based on the generated Abstract Syntax Trees (AST) of sub-queries to generate logical formulas, according to the formalism described in Section 3.4. In the case of θ-JOIN operation, DBverify first extracts the logical formula from the AST of the condition θ, and then adds it with the logical formula of the other part of the query. In the case of outer-JOIN, UNION, INTERSECT, and MINUS operations, we follow the formalism described in Section 3.4.

6. Experimental Evaluation

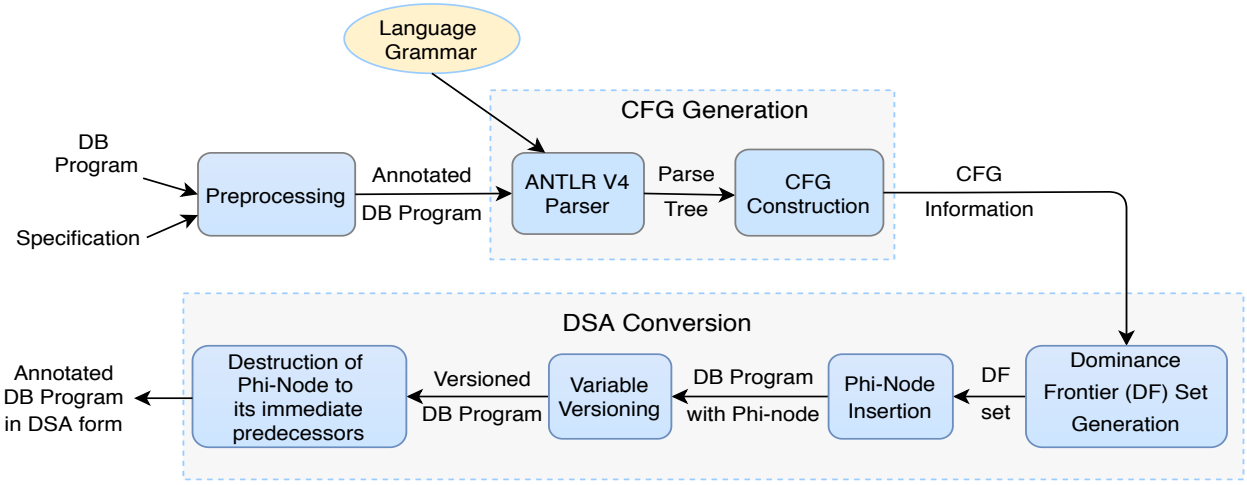
This section presents experimental results on a set of PL/SQL benchmark codes [21, 22, 23, 24, 25, 26, 27, 28] using our prototype tool DBverify. A summary of the benchmark codes is depicted in Table 4.

We organize our experimental reports in two subsections. In first, we assess the performance of deductive-based verification (symbolic execution, conditional normal form, and weakest precondition) techniques, and in the second we discuss about electing the most efficient algorithm. All experiments are conducted using a computer system equipped with core i7, 3.60 GHz CPU, 4GB memory, and Ubuntu 14.04 operating system.

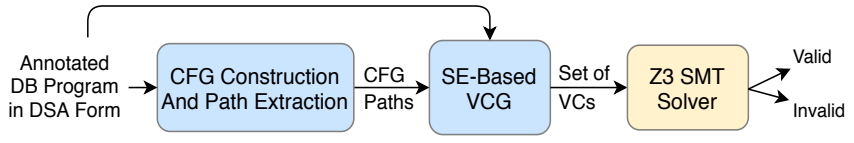
6.1. Assessing the Deductive-based Verification Techniques

DBverify accepts PL/SQL code annotated with assertions (representing properties of interest) as input, and it generates a set of VCs expressed in Z3 Language. The validity of VCs is then checked by providing their *negation* to Z3. The result `Unsat` indicates that the program satisfies its properties, whereas the result `Sat` for at least one case indicates a counter-example. For example, Z3 reports Sat for the VC (in negation form) generated from the procedure "budget" using weakest precondition and exhibits a model "[X = 1/2, CS = 5001, MP = 160001/2, DID = 0, EQ = 60001, CT = 10001, Z = 0, TA = 5/2, Y = 0]". This model serves as a counter-example for the correctness of the "budget" procedure.

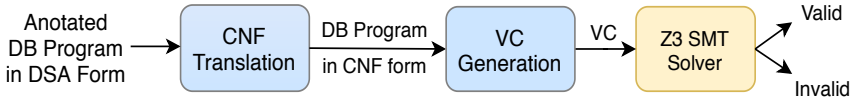
Table 5 depicts detailed verification results of the bench-



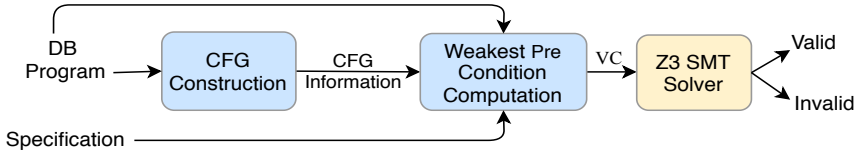
(a) Module `DSA-translator`



(b) Module `SE-verify`



(c) Module `CNF-verify`



(d) Module `wp-verify`

Figure 21: Schematic diagram of DBverify

mark applications under three deductive approaches. The procedures defined under each procedure provide various services and they are verified at the individual level as they are independent w.r.t. each other.

Since our special focus in this experiment is to verify database properties, the third and fourth columns of the table denote the number and type of assertions each procedure is instrumented with. The assertions with which we have annotated the PL/SQL procedures are either defined as part of the table definitions or we have chosen based on their practical relevance w.r.t. the procedures' behaviours. The assertion types are indicated by numbers, as follows: T1: Attribute-based, T2: Tuple-based, T3: Null Value, T4: Aggregate Functions, T5: General Assertions. The fifth column indicates the number of verification conditions (VCs) generated from the procedures under Symbolic Execution-based (SE) verification. Observe that Conditional Normal Form (CNF) and Weakest Precondition (WPC) always generate single VC. We have recorded total verification time (in millisec) for all procedures under three approaches in columns 6-8, and their comparison is depicted in Figure 22. This is worthwhile to observe that verification under CNF always takes less time as compared to SE and WPC. Interestingly, in the case of our benchmark codes, we also experience that SE requires more verification time as compared to WPC in most of the cases. Arguably, the reason behind this is the generation of multiple VCs (which may contain multiple copies of the same logical encoding) in SE and their validity checking by Z3 individually (depicted in Figure 27). However, an exception is observed in two procedures, namely P3 in IM and RM applications, where more VC generation time is experienced (depicted in Figure 24) due to the presence of more number of attributes in the postconditions as well as more number of SQL statements which define these attributes. Precisely, this affects the computation time from postcondition to weakest preconditions in the backward di-

Table 4
Description of benchmark database applications

DB applications	Description	LOC	#Procedures	#Attributes	#Application Variables
Courier_Company (CC) [21]	Application to Manages courier related information	935	16	40	122
Inventory_Management (IM) [22]	Maintain auto-part and assembling information	550	05	37	73
CableCity(CB) [23]	Manages database of stocks, sales and customer information	890	09	25	54
Hotel_Reservation (HR) [24]	Maintains details of guest, rooms, reservation, etc	504	05	19	24
Retail_Business Management (RM) [25]	Web application to keep records of sales of a retailer	552	04	35	24
Computer Store Management (CS) [26]	Application to manage the quantity, price, product details, etc. information of computer hardware store	738	05	38	34
Banking Management (BS) [27]	Application that manages information about accounts, debit, credit transaction details, etc. of customers.	511	04	23	34
Course Registration (CR) [28]	Application to maintain the course registration details of students in university	268	01	31	15

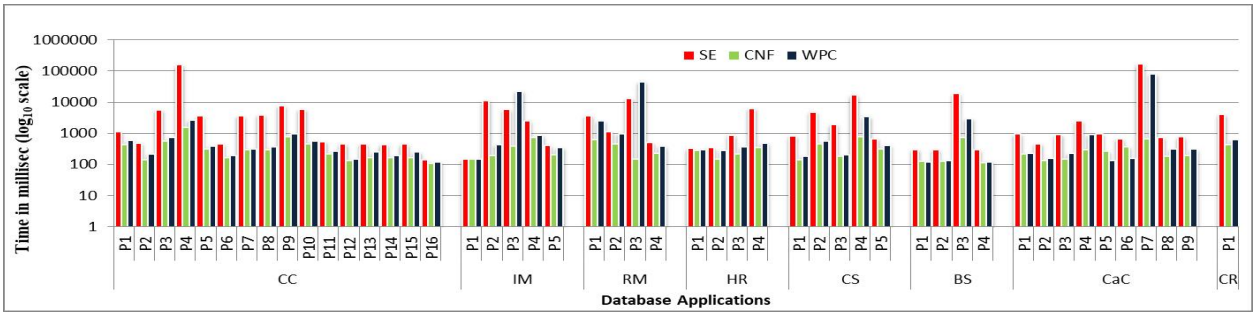


Figure 22: Total verification time of database procedures under deductive approaches

rection significantly. The verification outcomes indicating the number of assertions proved as valid and invalid are reported in columns 9 and 10 respectively. The results clearly show that only 38% of the benchmark procedures satisfy the annotated database properties, while 62% procedures violate either all or part of the annotated properties. The primary cause behind this is that most of the SQL statements in the procedures accept runtime inputs without any proper checking. In other way, only 68% assertions are satisfied by benchmark procedures.

Let us now draw a few other crucial observations based on our experimental results:

(I) DSA Generation (DSAgen) Time: Conversion of PL/SQL codes into their equivalent DSA form is a key step in SE- and CNF-based verification. Intuitively, various factors such as number and types of statements in the code, number of variables and attributes defined or used by the statements, number of conditions and nesting depth, etc., contribute differently in DSAgen time. To extract the insightful observation on the variation of DSAgen time w.r.t. the PL/SQL codes, we have classified statements into three different categories *low*, *medium* and *high* effective (denoted l, m and h respectively) depending upon their contributions in DSA-gen time and computed the overall weights of PL/SQL codes according to the following equation:

$$W = \sum_{i=1}^{n} (\texttt{Weight}(S_i^x) + \alpha_i) \tag{13}$$

where n is the number of statements and S_i^x denotes i^{th} statement in the category $x \in \{l, m, h\}$. $\texttt{Weight}(S_i^x)$ returns the weight of S_i^x under its category x. α_i is an additional weight factor whose value reflects the complexity level of S_i^x, i.e. the number of attributes and variables used and defined in S_i^x. Notice that this factor is implementation-dependent. We have classified the statements as follows: *Low effective* category includes variables declaration, assume, assert, cursor operations and exception handling statements. *Medium effective* category includes assignment statements. *Most effective* category include conditional and SQL statements. Figure 23 depicts the variation of DSAgen time w.r.t. the weights of the codes, considering $\texttt{Weight}(S_i^l) = 1$, $\texttt{Weight}(S_i^m) = 2$, and $\texttt{Weight}(S_i^h) = 4$.

(II) VC Generation (VCgen) Time: VCgen time for all procedures under SE, CNF and WPC are depicted in Figure 24. As expected, VCgen in CNF always takes less time as compared to the others. However, if we compare VCgen in SE and WPC, we observe that for some procedures SE performs better than WPC, while for the rest WPC performs better than SE. Although

Table 5
Verification results under deductive-based approaches

DB App	Procedures	# Assertion	Assertion Type	# VCs(SE)	Verification Time (in millisec)			Verification Results	
					SE	CNF	WP	#Valid	#Invalid
CC	update_client.sql (P1)	3	T1,T3	7	1132.382	429.915	605.012	2	1
	add_dimension_class.sql (P2)	1	T1,T3	3	489.062	141.523	223.742	1	0
	add_car.sql (P3)	2	T1-T4	36	5715.589	571.418	729.936	1	1
	add_courier.sql (P4)	7	T1-T4	726	158809.247	1610.636	2652.89	6	1
	add_status.sql (P5)	4	T1,T3	24	3623.771	317.157	397.077	4	0
	add_parcel_type.sql (P6)	1	T1,T3,T4	3	456.85	170.738	193.391	1	0
	add_client.sql (P7)	2	T1,T3,T4	18	3681.689	305.573	323.188	0	2
	add_delivery_attempt.sql (P8)	4	T1,T3	27	3967.09	296.309	379.003	3	1
	add_parcel.sql (P9)	3	T1,T3,T4	40	7840.22	776.464	972.306	3	0
	add_warehouse.sql (P10)	2	T1,T3	24	6005.307	473.43	576.8599	2	0
	update_warehouse.sql (P11)	3	T1,T3	3	553.536	213.336	270.883	2	1
	driving_license_category (P12)	1	T1,T3	3	450.966	134.064	152.183	1	0
	get_contract_type_id.sql (P13)	1	T1,T3	3	469.39	170.085	253.615	1	0
	get_delivery_status.sql (P14)	1	T1,T3	3	444.132	163.679	197.397	1	0
	get_country_id.sql (P15)	1	T1,T3	3	450.762	171.882	256.791	1	0
	delete_client.sql (P16)	1	T1	7	141.092	107.117	119	1	0
IM	add-to-inventory.sql (P1)	1	T1	1	150.933	148.536	150.597	1	0
	assemble-module.sql (P2)	4	T1,T2	30	5822.301	195.399	437.9089	0	4
	assemble-component.sql(P3)	10	T1-T5	78	11235.171	397.668	22882.824	8	2
	procinventory.sql (P4)	7	T1,T3,T5	15	2577.703	730.68	854.946	6	1
	deliver.sql (P5)	3	T1,T3	3	405.958	205.872	345.739	2	1
RM	add_cust.sql (P1)	2	T1,T3,T5	9	3740.924	620.314	2561.225	0	2
	qoh_update.sql(P2)	4	T1,T3	7	1115.73	458.236	989.809	1	3
	retail-business-logic.sql (P3)	3	T1,T2,T5	72	13464.077	153.664	45707.734	2	1
	budget.sql (P4)	1	T1,T2,T5	2	510.087	231.96	393.028	0	1
HR	bill.sql (P1)	1	T1,T5	2	332.12	278.318	303.683	0	1
	award-bonus.sql (P2)	1	T2	2	347.44	148.783	291.376	0	1
	discount.sql (P3)	1	T1,T5	4	897.481	217.413	369.509	0	1
	resrvation-proc.sql (P4)	5	T1,T3	30	6427.385	348.296	488.749	4	1
CS	cust-emp-proc.sql (P1)	2	T1,T2	6	850.02	141.256	189.866	2	0
	cartsell.sql (P2)	5	T1-T3	30	4944.753	454.442	567.582	5	0
	loginproc.sql (P3)	2	T1-T3	9	1951.349	188.744	206.902	0	2
	product.sql (P4)	4	T1,T3	79	17081.274	770.027	3453.8509	3	1
	transfer.sql (P5)	4	T1,T3	4	686.796	314.784	416.193	4	0
BS	credit-account.sql (P1)	1	T1	1	305.4	129.629	121.744	1	0
	debit-account.sql (P2)	1	T1,T5	2	302.246	127.123	134.39	0	1
	Procedure_transactions.sql (P3)	7	T1,T3,T4	64	19009.815	749.565	3019.334	6	1
	check.sql (P4)	1	T1	2	294.594	118.144	123.658	0	1
CaC	AddCustomerPoints.sql (P1)	1	T1	4	952.831	222.605	232.99	0	1
	CheckPassword.sql (P2)	1	T1,T2	3	469.283	132.547	154.568	1	0
	UpdateQuantity.sql (P3)	1	T1,T2	6	902.813	150.781	233.769	1	0
	RecordNewSale.sql (P4)	1	T1	16	2544.719	307.056	911.324	0	1
	PopulateProducts.sql (P5)	1	T1,T3	4	974.6	264.795	135.582	0	1
	PopulateCustomers.sql (6P)	1	T1,T3	4	678.47	377.758	162.135	0	1
	PopulateSales.sql (P7)	4	T1-T5	1024	169602.727	662.117	80132.817	3	1
	DecreaseDispStock.sql (P8)	1	T1	5	765.373	190.366	313.94	0	1
	IncreaseDispStock.sql (P9)	1	T1	5	782.142	197.925	312.692	0	1
CR	isEnrollable.sql (P1)	1	T1,T3	14	4164.848	434.723	634.9329	1	0

VC generation in both approaches gets highly affected by the presence of conditional statements, the primary reason behind this difference in VCgen time is as follows: In SE, the logical encoding of all statements along each path appears in its corresponding VC. This creates multiple copies of the logical encoding of the same statement in multiple VCs, if the statement appears in all those paths. Therefore, procedures having a large number of execution paths which tolerate multiple copies of the same statements' logical encoding experience higher VCgen time (for example, procedures P4, P9 and P10 in CC, P2 in IM, P4 in RH,

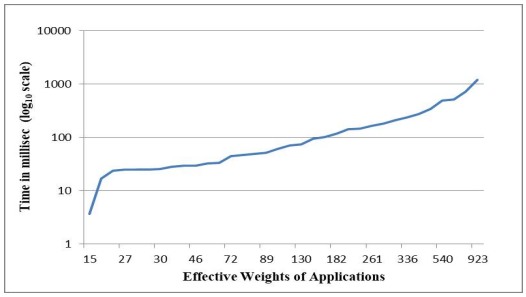


Figure 23: DSAgen Time VS Effective Weights of Procedures

and P3 in BS). In contrast, in WPC, VC generation deals with the computation of the weakest precondition from the postcondition in a backward direction and this primarily gets affected by only those statements which define the attributes or variables that appeared in the postcondition. In particular, SQL statements as defining statements are more influential in this case, as they involve `WHERE` clause. Therefore, procedures having more such defining statements experience higher VCgen in WPC (for example, P3 in IM and RM, P4 in CS, etc.).

Let us now observe the variation of VCgen w.r.t. PL/SQL codes, identifying the influence of different types of statements and their operational complexity to the VCgen. The computation of weights of PL/SQL codes in case of SE, CNF and WPC follows similar approach as in the case of DSAgen time, with minor changes either in equation or in statements' classification. In SE and CNF, conditional statements are considered as *medium* category, whereas WPC considers the assert and conditional statements as *high* category. Although the weight computation in CNF and WPC follows Equation 13, it differs in case of SE by taking into account various paths in the code as defined in Equation 14.

$$W = \sum_{r=1}^{p} \Big(\sum_{i=1}^{n} (\texttt{Weight}(S_{i,r}^{x}) + \alpha_{i,r}) \Big) \tag{14}$$

Following the Equations 13 and 14, the variation of VCgen time w.r.t. weights of different procedures in the case of SE, CNF, and WPC are depicted in Figures 25(a), 25(b), and 25(c) respectively.

(III) Number of VCs: The number of VCs in the SE-based technique is same as the number of execution paths that exist in the code, and it depends on the number of conditions and their nesting structure. the presence of conditional statements without any nesting in the code yields a maximum number of paths, whereas a balanced form of nesting in both if-block and else-block yields a minimum number of paths in the code. Therefore, given n number of conditional statements in the code, the number of paths (hence VCs) lie within the interval $[n+1, 2^n]$. We observe that our results on the number of generated VCs lie between this allowable range depicted in Figure 26.

(IV) Z3 Execution time (Z3exe): Since we have built our tool on the top of SMT solver Z3, the time taken by the Z3 to validate the generated formula for all procedures under three deductive approaches is depicted in Figure 27. This is observed that Z3 execution time for the VCs generated in CNF always takes less time compared to others. However, in the case of SE and WPC, is depends on the number of VCs generated in SE versus the size of VC generated in WPC.

6.2. Electing Most Efficient Verification Algorithm

While evaluating different deductive verification algorithms with a common goal, the first question comes to mind is "which one is the most efficient or suitable algorithm?". Let us address this question by analyzing the results from both theoretical and practical perspectives.

As already discussed in Section 4, for a given program with n conditional statements, there exist $O(2^n)$ execution paths and therefore SE-based algorithm results into $O(2^n)$ number of VCs. In contrast, the CNF algorithm avoids path enumeration by transforming the procedure into CNF-form. Assuming that the program contains n conditional statements with nesting depth up to m, the CNF algorithm generates a single VC of size $O(n + m^2)$. WPC algorithm, on the other hand, generates single VC by OR-ing the weakest preconditions generated from the postcondition along all branches of the code. This exhibits exponential $O(2^n)$ size of VC for programs with n conditional statements.

Let us now do an analysis based on our empirical study. As clear in Figures 24, CNF always takes less VC generation time as an evidence to the theoretical analysis results. However, VCgen in case of SE and WPC, although both are influenced by conditional statements, varies due to the presence of other statements. In particular, as already mentioned before, SE performs path enumeration and takes into account the same statements multiple times during the VC generation, if the statements appear over multiple paths. This consumes a significant amount of time which varies depending upon the number of paths and their length. On the other hand, VC generation in WPC effectively depends on the computation of weakest precondition from postconditions. This consumes a significant amount of time which increases if postcondition involves more number of attributes and variables and there exist more number of statements which define those attributes or variables. Although, our empirical study does not allow us to quantify these factors exactly, one can conclude that programs having less number of defining statements and more number of execution paths experience more VCgen time in SE than that in case of WPC.

A similar observation can also be drawn from the Z3 execution time, depicted in Figure 27, where VCs generated in CNF always takes less time as compared to the execution of VCs generated in SE and WPC. However, we observe in most of the cases that Z3 execution time in SE is more than that in WPC, due to the presence of multiple VCs (with rep-

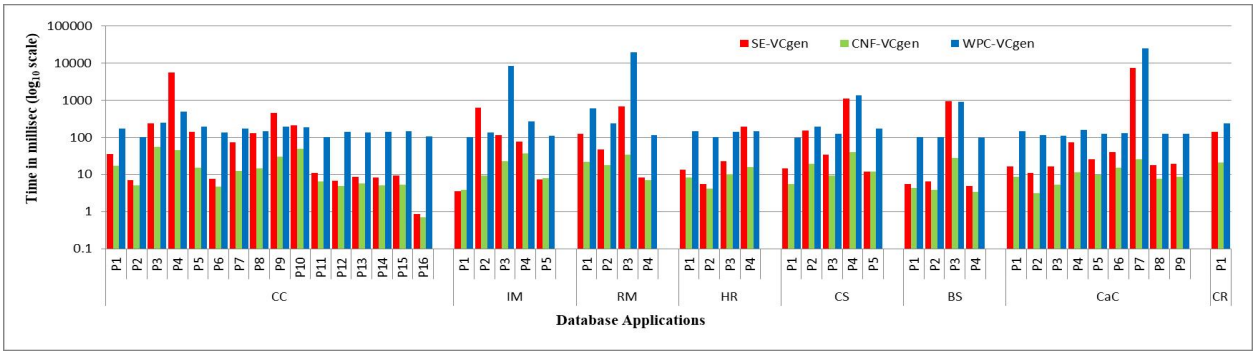
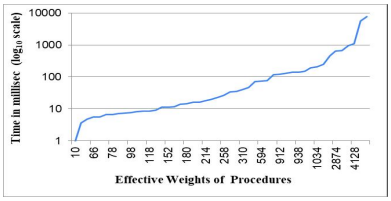
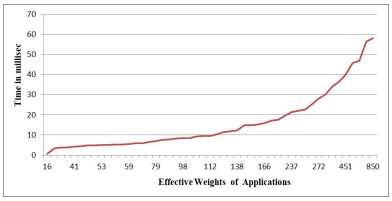


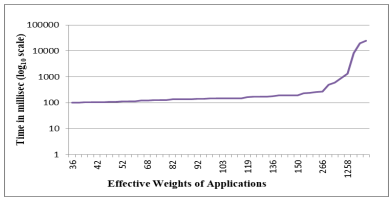
Figure 24: VC Generation (VCgen) Time under deductive approaches



(a) SE-VCgen Time VS Effective Weights of Procedures



(b) CNF-VCgen Time VS Weight of Effective of of Procedures



(c) WPC-VCgen Time VS Weight of Effective of Procedures

Figure 25: VCgen Time under deductive approaches

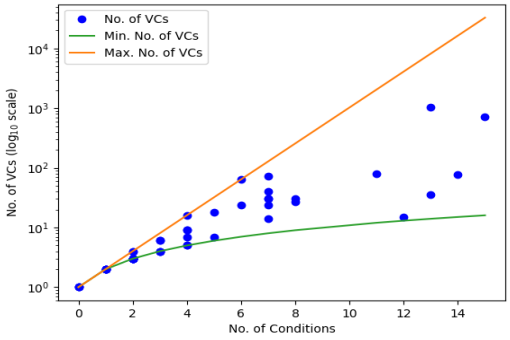


Figure 26: Number of VCs generated in SE

etition of logical encoding) and their validity checking individually. Exception is observed in two cases, namely P3 in IM and RM, where the size of VC generated in WPC is much higher than the total size of all VCs generated in SE. This happens due to the presence of more number of attributes in postcondition and more number of SQL statements which define those attributes. As WPC does not require DSA conversion, this becomes a key factor on the overall verification time depicted in Figure 22. Based on this analysis, one can now easily judge that CNF is the most efficient verification algorithm among these three. However, the decision to choose one between SE and WPC depends on the program's structure, size of the postcondition and the types of statements involved in the program.

7. Threats to Validity

We first discuss the threats to external validity, which are about the generalization of our findings. Three proposed VC generation techniques in this paper are, in general, applicable to the case of database applications dealing with relational databases. In particular, our focus is to verify their correctness w.r.t. database properties. In terms of expressive power of the assertion language, although we are able to express most common database properties [19], the failure is observed in case of property that refers to referential integrity or involves the count of database records. Notably, our theoretical formalism is based on the abstract syntax of a database language embedded within an imperative host language. Although this paper considers a simple form of host imperative language, the support of dynamic memory data structures, floating points, pointers, etc., can be provided with more engineering without affecting the general idea of the proposed techniques. Besides, the abstract syntax of database counter-part captures crucial features of the languages used in popular structured database systems, such as MySQL, Oracle, DB2, Microsoft SQL Server, PostgreSQL, etc., with a complete support for data manipulation operations. It should be mentioned that the proposed approach does not support verification of dynamically generated database statements in the applications. The developed tool DBverify currently supports loop-free PL/SQL only. We are in the process of extending its supports to loop (as highlighted in Section 3.5) and to the languages of other database management systems with their embedding within popular host languages (such as C, Python, Java, etc.) in the next re-

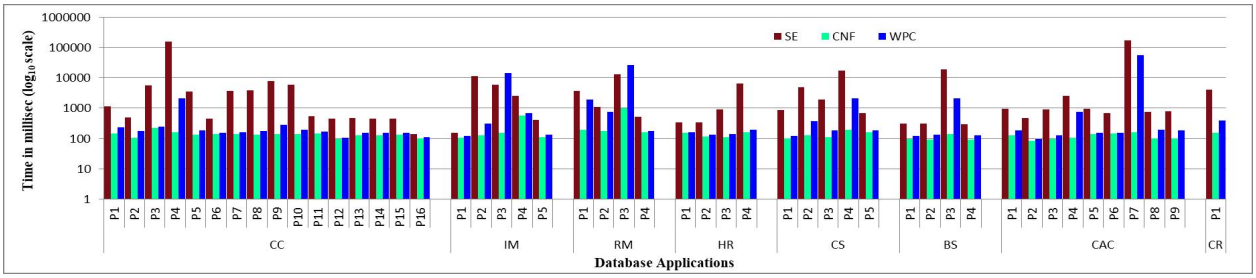


Figure 27: Z3 Execution Time of deductive approaches

lease of the tool. For the latter case, we require little effort to modify the parser-based language processing module in DBverify according to the language's concrete syntax. Observe that, in the case of concurrent database transactions, as two or more transactions can interleave in a concrete program run, this threat can be mitigated by our current proposal through a static identification of all possible permutations of database statements present in the transactions [48, 49], incurring an exponential computation cost [50, 51].

Let us now discuss the threats to internal validity, which refer to experimental bias and errors. In our experiment, we have annotated benchmark PL/SQL procedures with the assertions that are already part of the underlying database table definitions or chosen based on their practical relevance w.r.t. the procedures' behavior. To this aim, we consider the most common relational database properties, as reported in [19]. Even though our proposal covers crucial SQL features, the aggregate functions (except COUNT) are treated with an abstraction and this may results in false positives. The experimental results, as expected, depict that CNF is the most efficient verification algorithm among these three. However, the decision to choose one between SE and WPC depends on the program's structure, size of the postcondition and the types of statements involved in the program.

8. Related Works

Theorem proving (Deductive reasoning) [2, 3, 4], model checking (algorithmic verification) [5, 6, 7] and process algebra [8] are the main categories of techniques for formally verifying properties of both hardware and software systems. Theorem proving and model checking approaches have complementary strengths and weaknesses, and their combination promises to enhance the capabilities of each [52, 53]. Process algebra, on the other hand, constitutes a framework for formal verification of processes and data, with the emphasis on processes that are executed concurrently.

Over the decades, numerous proposals based on the above-mentioned methods are introduced in the literature for the verification of general purpose programming languages, addressing various language features such as variables, control structures, pointers, objects, etc. [2, 3, 4, 10, 41]. In addition, database researchers have also shown significant interest in verifying database applications using theorem proving [14, 15, 16, 17, 18] and model checking [54, 55, 56, 57, 58, 59, 60]. Apart from this, there has also been some proposal on testing [61, 62, 63, 64], equivalences checking [65, 66], schema refactoring [67, 68] as well as synthesis [69, 70].

The authors in [17] propose an approach for verification of web application embedded with SQL. This requires the translation of embedded SQL scripts into SmpSL functions followed by the computation of verification conditions of SmpSL functions using *weakest precondition*. The purpose is to verify integrity constraints defined on the underlying database. The limited expressibility of SmpSL does not cover aggregate functions or arithmetic operations. Integrity constraints verification in the object-oriented database programming language O_2 is proposed in [15]. For a given method m and constraint C, m can not violate C if $\overrightarrow{\mathrm{m}}(C) \Rightarrow C$ or $C \Rightarrow \overrightarrow{\mathrm{m}}(C)$, where $\overrightarrow{\mathrm{m}}(\phi)$ is a postcondition for a given precondition ϕ and $\overleftarrow{\mathrm{m}}(\phi)$ is a precondition for a given postcondition ϕ. The proposed approach computes verification conditions using either *weakest precondition* or *strongest postcondition* computations. Authors in [16] propose integrity constraints verification for database applications using transformation operators. The proposed approach expressed every update operation as a predicate $\mathbf{U} = P(\vec{a})$, where $\vec{a}$ denotes a sequence of constants. Integrity constraints are defined in a constraint theory τ. The function $\mathbf{after}^U(\tau)$ translates the constraint theory to the weakest precondition of ϕ with respect to the update $\mathbf{U}$ and a simplified formula is obtained by applying function $\mathbf{Simp}^U(\phi)$. The resultant formula is executed as a query and if it returns empty then the database is consistence, otherwise the returned tuple provides hints for extending the update in order to restore the consistency. Verification of database integrity constraints using refinement types is proposed in [14]. The proposed tool maps an SQL schema S to a Stateful F7 module $[\![S]\!]$ by using a sequence of type definitions, predicate definitions, and function signatures. User transactions are written using the functional language F#. Verification of integrity constraints starts by generating a set of verification conditions from F# and SQL codes, which are then passed to an automatic theorem prover. Unfortunately, the syntax of SQL considered in the proposed work does not include nested queries or aggregate functions. A fully verified in-memory relational database management system (RDBMS) using Coq has been proposed in [18]. The authors mainly verify whether or not the RDBMS correctly executes queries

w.r.t. the denotational semantics of SQL and relations.

Verification of the functional correctness of database-driven applications using model checking is proposed in [58]. The author introduces the WAVE tool, which allows the users to specify functional correctness properties using LTL formulas and verify a given database-driven application against these functional properties. Authors in [54] consider the verification of monadic second-order properties of runs in a model where the underlying database can be updated by insertion or deletion. Decidability is obtained for recency bounded artifacts, in which only recently introduced values are retained in the current data. A theoretical approach for the verification of database-driven systems is proposed in [55], where the authors use symbolic model-checking via model completions (equivalently, via *covers*). Interesting works on real-time and distributed systems using temporal logic to obtain verification models are proposed in [71, 72]. The correctness verification of service composition methods in a multi-cloud computing environment based on event-based QoS factors is presented in [73].

Our work draws motivation from [13, 29, 30]. In [13, 29, 30], the authors describe in detail all three approaches, namely symbolic execution, conditional normal form, and weakest precondition, to generate VCs for imperative languages. Our work can be seen as an extension of the same to the case of database applications.

9. Conclusions

This work contributes towards the verification of database applications by proposing a set of comprehensive techniques to generate Verification Conditions from database programs. With respect to the literature, the proposed approach shows its competence to support crucial SQL features along with its embedding into other host imperative language and allows the verification of common database properties. We develop DBverify, a verification tool implemented in python based on our theoretical foundation, which enables users to verify PL/SQL procedures under three different approaches. The detailed performance analysis based on the experimental results on a set of benchmark PL/SQL codes demonstrates the effectiveness of the approaches under various circumstances. Notably, the performance of the CNF algorithm is observed better than the other two approaches in all the cases. For the given set of PL/SQL codes with chosen properties, the experimental results show that only 38% of the benchmark procedures satisfy the annotated database properties, while 62% procedures violate either all or part of the annotated properties. The primary cause for the latter case is mostly due to the acceptance of runtime inputs in SQL statements without any proper checking.

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A. Appendix

A.1. Proofs

Lemma 2. *Let Φ be the logical encoding of an execution path π up to program point ℓ. Let $stmt^{dsa}$ be a DSA form of program statement at program point $\ell+1$. Let Ψ be the logical encoding computed using the function* Path(Φ, $stmt^{dsa}$). *The function* Path *is correct if* $\forall\ (\rho_d,\ \rho_a)\ :\ (\rho_d,\ \rho_a) \vDash_I \Phi$ *under interpretation I and $\mathcal{T}_{dba}[\![stmt^{dsa}]\!](\rho_d,\ \rho_a) = (\rho_{d'},\ \rho_{a'})$, implies $(\rho_{d'},\ \rho_{a'}) \vDash_I \Psi$.*

Proof. The proof is established based on structural induction. Assume that $(\rho_d,\ \rho_a)$ satisfies Φ under the interpretation I, i.e. $(\rho_d,\ \rho_a) \vDash_I \Phi$.

- Assignment Statement $v_{i+1} := e_i$. Let $\mathcal{T}_{dba}[\![v_{i+1} := e_i]\!](\rho_d, \rho_a) = (\rho_{d'}, \rho_{a'})$, where $(\rho_{d'}, \rho_{a'})$ is obtained by substituting all occurrences of v_{i+1} in (ρ_d, ρ_a) by $\mathcal{T}_{dba}[\![e_i]\!](\rho_d, \rho_a)$. Since e_i does not involve v_{i+1}, its semantics are same w.r.t. both the states (ρ_d, ρ_a) and $(\rho_{d'}, \rho_{a'})$. Therefore,

  $$(\rho_{d'}, \rho_{a'}) \vDash_I (v_{i+1} == e_i) \tag{15}$$

  On the other hand, since v_{i+1} is not present in Φ due to DSA property and only v_{i+1} is affected in $(\rho_{d'}, \rho_{a'})$, we have

  $$(\rho_{d'}, \rho_{a'}) \vDash_I \Phi \tag{16}$$

  Combining Equations 15 and 16, we get $(\rho_{d'},\ \rho_{a'}) \vDash_I (\Phi \wedge v_{i+1} == e_i)$.

- Database Statements Q. Since a database statement Q involves action 'A' and condition 'cond', this can be treated as a guarded command equivalent to "if *cond* then A". Consider the DSA form $\langle \vec{a}_{i+1} := \mathsf{act}_i^1 \curvearrowright \mathsf{act}_i^2,\ cond_i^1 \curvearrowright cond_i^2 \rangle$ of different database statements. By the definition of the function Path, we have:

  $$\begin{aligned}\mathsf{Path}(\Phi,\ \langle \vec{a}_{i+1} := \mathsf{act}_i^1 \curvearrowright \mathsf{act}_i^2,\ cond_i^1 \curvearrowright cond_i^2 \rangle) = \\ \mathsf{Path}(\Phi, \langle \vec{a}_{i+1} := \mathsf{act}_i^1,\ cond_i^1 \rangle) \vee \\ \mathsf{Path}(\Phi, \langle \vec{a}_{i+1} := \mathsf{act}_i^2,\ cond_i^2 \rangle)\end{aligned}$$

  Let us now prove the lemma for each database action:

  - $Q_{upd}^{dsa} \triangleq \langle \vec{a}_{i+1} := \mathtt{UPDATE}(\vec{a}_i) \curvearrowright \mathtt{UPDATE}(\vec{e}_i),\ \neg cond_i \curvearrowright cond_i \rangle$. Given a state (ρ_d, ρ_a), assume that $\mathcal{T}_{dba}[\![Q_{upd}^{dsa}]\!](\rho_d, \rho_a) = (\rho_{d'}, \rho_{a'})$. By the definition of the function Path, we have

    $$\Psi = \mathsf{Path}(\Phi,\ Q_{upd}^{dsa}) = \Phi \wedge ((\neg cond_i \wedge \bigwedge_{j=1}^{|\vec{a}_{i+1}|} a_{i+1}^j == a_i^j) \vee (cond_i \wedge \bigwedge_{j=1}^{|\vec{a}_{i+1}|} a_{i+1}^j == e_i^j))$$

    Since $(\rho_d, \rho_a) \vDash_I \Phi$ and Φ does not involve $\vec{a}_{i+1}$ due to DSA property, the following holds

    $$(\rho_{d'}, \rho_{a'}) \vDash_I \Phi \tag{17}$$

    Now we have two possibilities: either $(\rho_{d'}, \rho_{a'}) \vDash_I \neg cond_i$ or $(\rho_{d'}, \rho_{a'}) \vDash_I cond_i$. In the former case, the action $\vec{a}_{i+1} := \mathtt{UPDATE}(\vec{a}_i)$ results into the same semantics for $\vec{a}_i$ and $\vec{a}_{i+1}$ w.r.t. $(\rho_{d'}, \rho_{a'})$, i.e. $\mathcal{T}_{dba}[\![\vec{a}_i]\!](\rho_{d'}, \rho_{a'}) = \mathcal{T}_{dba}[\![\vec{a}_{i+1}]\!](\rho_{d'}, \rho_{a'})$. Therefore,

    $$(\rho_{d'}, \rho_{a'}) \vDash_I (\neg cond_i \wedge \bigwedge_{j=1}^{|\vec{a}_{i+1}|} a_{i+1}^j == a_i^j) \tag{18}$$

    In the latter case, the attributes $a_{i+1}^j \in \vec{a}_{i+1}$ in $(\rho_{d'}, \rho_{a'})$, where $j = 1 \ldots |\vec{a}_{i+1}|$, are substituted by $\mathcal{T}_{dba}[\![e_i^j]\!](\rho_d, \rho_a)$ respectively. Since only the value of $\vec{a}_{i+1}$ is affected in $(\rho_{d'}, \rho_{a'})$, we have $\mathcal{T}_{dba}[\![\vec{a}_{i+1}]\!](\rho_{d'},\ \rho_{a'}) = \mathcal{T}_{dba}[\![\vec{e}_i]\!](\rho_{d'},\ \rho_{a'})$, and therefore

    $$(\rho_{d'}, \rho_{a'}) \vDash_I (cond_i \wedge \bigwedge_{j=1}^{|\vec{a}_{i+1}|} a_{i+1}^j == e_i^j) \tag{19}$$

    Hence, combining Equations 17, 18 and 19, $(\rho_{d'},\ \rho_{a'}) \vDash_I \Psi$ is proved.

  - $Q_{ins}^{dsa} \triangleq \langle \vec{a}_{i+1} := \mathtt{UPDATE}(\vec{a}_i) \curvearrowright \mathtt{INSERT}(\vec{e}_i),\ true \curvearrowright false \rangle$. Given a state $(\rho_d,\ \rho_a)$, assume that $\mathcal{T}_{dba}[\![Q_{ins}^{dsa}]\!](\rho_d,\ \rho_a) = (\rho_{d'},\ \rho_{a'})$. By the definition of the function Path, we have

    $$\Psi = \mathsf{Path}(\Phi,\ Q_{ins}^{dsa}) = \Phi \wedge ((\bigwedge_{j=1}^{|\vec{a}_{i+1}|} a_{i+1}^j == a_i^j) \vee (\bigwedge_{j=1}^{|\vec{a}_{i+1}|} a_{i+1}^j == e_i^j))$$

    Since $(\rho_d, \rho_a) \vDash_I \Phi$ and Φ does not involve $\vec{a}_{i+1}$ due to DSA property, we have

    $$(\rho_{d'},\ \rho_{a'}) \vDash_I \Phi \tag{20}$$

    According to the action $\vec{a}_{i+1} := \mathtt{UPDATE}(\vec{a}_i)$ in Q_{ins}^{dsa}, the semantics of $\vec{a}_i$ and $\vec{a_{i+1}}$ w.r.t. $(\rho_d,\ \rho_a)$ are same, i.e., $\mathcal{T}_{dba}[\![\vec{a}_i]\!](\rho_{d'},\ \rho_{a'}) = \mathcal{T}_{dba}[\![\vec{a}_{i+1}]\!]$ $(\rho_{d'},\ \rho_{a'})$. Therefore,

    $$(\rho_{d'},\ \rho_{a'}) \vDash_I (\bigwedge_{j=1}^{|\vec{a}_{i+1}|} a_{i+1}^j == a_i^j) \tag{21}$$

However, according to the second action $\vec{a}_{i+1}$:= INSERT($\vec{e}_i$), a single tuple containing values $\mathcal{T}_{dba}[\![e_i^j]\!](\rho_d, \rho_a)$ corresponding to attributes a_{i+1}^j where $j = 1 \dots |\vec{a}_{i+1}|$ is inserted into the database. Since only the value of $\vec{a}_{i+1}$ is affected in $(\rho_{d'}, \rho_{a'})$, we have $\mathcal{T}_{dba}[\![\vec{a}_{i+1}]\!](\rho_{d'}, \rho_{a'}) = \mathcal{T}_{dba}[\![\vec{e}_i]\!](\rho_{d'}, \rho_{a'})$, and therefore

$$(\rho_{d'}, \rho_{a'}) \vDash_I (\bigwedge_{j=1}^{|\vec{a}_{i+1}|} a_{i+1}^j == e_i^j) \tag{22}$$

Hence, combining Equations 20, 21 and 22, $(\rho_{d'}, \rho_{a'}) \vDash_I \Psi$ is proved.

– $Q_{del}^{dsa} \triangleq \langle \vec{a}_{i+1}$:= UPDATE($\vec{a}_i$) $\curvearrowright$ DELETE(), $\neg cond_i \curvearrowright cond_i \rangle$. Given a state (ρ_d, ρ_a), assume that $\mathcal{T}_{dba}[\![Q_{ins}^{dsa}]\!](\rho_d, \rho_a) = (\rho_{d'}, \rho_{a'})$. By the definition of the function Path, we have

$$\Psi = \mathsf{Path}(\Phi, Q_{del}^{dsa}) = \Phi \wedge (\neg cond_i \bigwedge_{j=1}^{|\vec{a}_{i+1}|} a_{i+1}^j == a_i^j)$$

Since $(\rho_d, \rho_a) \vDash_I \Phi$ and Φ does not involve $\vec{a}_{i+1}$ due to DSA property, the following holds

$$(\rho_{d'}, \rho_{a'}) \vDash_I \Phi \tag{23}$$

Like previous cases, we have two possibilities: either $(\rho_{d'}, \rho_{a'}) \vDash_I \neg cond_i$ or $(\rho_{d'}, \rho_{a'}) \vDash_I cond_i$. In the former case, similar to Equation 6, we have

$$(\rho_{d'}, \rho_{a'}) \vDash_I (\neg cond_i \wedge \bigwedge_{j=1}^{|\vec{a}_{i+1}|} a_{i+1}^j == a_i^j) \tag{24}$$

While in the latter case, tuples satisfying $cond_i$ are removed from the database table. Therefore, $(\rho_{d'}, \rho_{a'}) \vDash_I \Psi$ is proved.

– $Q_{sel}^{dsa} \triangleq \langle rs_{i+1}$:= SELECT($f(\vec{e}_i), r(\vec{h}(\vec{a}_i))$, ϕ', $g(\vec{e}_i)$), $cond_i \rangle$. Let $\mathcal{T}_{dba}[\![Q_{sel}^{dsa}]\!](\rho_d, \rho_a) = (\rho_{d'}, \rho_{a'})$. According to the Path function, we have Ψ = Path(Φ, Q_{sel}^{dsa}) = $\Phi \wedge ((cond_i \wedge rs_{i+1} = \mathsf{F}(\vec{a}_i)) \vee (\neg cond_i \wedge rs_{i+1} = rs_i))$. Following the similar direction as above, we can easily prove $(\rho_{d'}, \rho_{a'}) \vDash_I \Psi$ by taking the following facts into the consideration:

(a) The affected application variable rs_{i+1} is not involved in Φ, so $(\rho_{d'}, \rho_{a'}) \vDash_I \Psi$.

(b) When $(\rho_d, \rho_a) \vDash_I \neg cond_i$ then $(\rho_{d'}, \rho_{a'}) \vDash_I (\neg cond_i \wedge rs_{i+1} = rs_i)$

(c) When $(\rho_d, \rho_a) \vDash_I cond_i$ then $(\rho_{d'}, \rho_{a'}) \vDash_I (cond_i \wedge rs_{i+1} = \mathsf{F}(\vec{a}_i))$.

□

Theorem 5. *Given an annotated program {assume ϕ_1; $\mathcal{P}$; assert ϕ_2;}. If the verification condition $\phi_1 \implies wp(\mathcal{P}, \phi_2)$ is valid, then $\mathcal{P}$ satisfies the property ϕ_2 and vice versa.*

Proof. The proof is based on structural induction on $stmt \in \mathcal{P}$. Recall the definition of wp in Figure 16 and let wp($stmt$, ψ_2) = ψ_1. We have to prove that, if $(\rho_d, \rho_a) \vDash_I \phi_1$ and $\phi_1 \rightarrow \psi_1$ and $\psi_2 \rightarrow \phi_2$ and $\mathcal{T}_{dba}[\![stmt]\!](\rho_d, \rho_a) = (\rho_{d'}, \rho_{a'})$, then $(\rho_{d'}, \rho_{a'}) \vDash_I \psi_2$. Let us now consider the different cases:

- Assignment $\triangleq v := e$. Let $(\rho_d, \rho_a) \vDash_I \phi_1$. According to the definition, wp($v := e$, ψ_2) = $\psi_2[e/v]$. Assume that $\phi_1 \rightarrow \psi_2[e/v]$. Therefore,

$$(\rho_d, \rho_a) \vDash_I \psi_2[e/v] \tag{25}$$

According to the transition semantics, assume $\mathcal{T}_{dba}[\![v := e]\!](\rho_d, \rho_a) = (\rho_{d'}, \rho_{a'})$ such that

$$\rho_a(z) = \begin{cases} m, \ if \ z = v \\ \rho_a(z), \ Otherwise \end{cases}$$

where $m = \mathcal{T}_{dba}[\![e]\!](\rho_d, \rho_a)$. Since, in $(\rho_{d'}, \rho_{a'})$ all occurrences of x are replaced by m, according to Equation 25, we have $(\rho_{d'}, \rho_{a'}) \vDash_I \Psi_2$. Assuming $\psi_2 \rightarrow \phi_2$, this case is proved.

- $Q_{upd} \triangleq \langle \vec{a}$:= UPDATE($\vec{e}$), $cond \rangle$. Let $(\rho_d, \rho_a) \vDash_I \phi_1$. According to the definition, wp($\langle \vec{a}$:= UPDATE($\vec{e}$), $cond \rangle$, ψ_2) = $((\psi_2 \wedge \neg cond) \vee (\psi_2[\vec{e}/\vec{a}] \wedge cond))$. Assume that $\phi_1 \rightarrow ((\psi_2 \wedge \neg cond) \vee (\psi_2[\vec{e}/\vec{a}] \wedge cond))$. Therefore,

$$(\rho_d, \rho_a) \vDash_I ((\psi_2 \wedge \neg cond) \vee (\psi_2[\vec{e}/\vec{a}] \wedge cond)) \tag{26}$$

According to the semantics function, let us assume $\mathcal{T}_{dba}[\![Q_{upd}]\!](\rho_d, \rho_a) = (\rho_{d'}, \rho_{a'})$, where

(i) Values of $\vec{a}$ will be updated by $\mathcal{T}_{dba}[\![\vec{e}]\!](\rho_d, \rho_a)$ for the tuples satisfying '$cond$'. That is, like assignment statement, we have

$$(\rho_{d'}, \rho_{a'}) \vDash_I \Psi_2 \ when \ (\rho_d, \rho_a) \vDash_I \Psi_2[\vec{e}/\vec{a}] \wedge cond \tag{27}$$

(ii) Values of $\vec{a}$ will remain unchanged for those tuples which do not satisfy '$cond$'. That is,

$$(\rho_{d'}, \rho_{a'}) \vDash_I \Psi_2 \ when \ (\rho_d, \rho_a) \vDash_I \Psi_2 \wedge \neg cond \tag{28}$$

Combining Equations 26, 27 and 28, the lemma is proved for Q_{upd} assuming $\Psi_2 \rightarrow \phi_2$.

- $Q_{ins} \triangleq \langle \vec{a}$:= INSERT($\vec{e}$), $false \rangle$. According to definition of wp, wp($\langle \vec{a}$:= INSERT($\vec{e}$), $false \rangle$, ψ_2) = $(\psi_2[\vec{e}/\vec{a}] \vee \psi_2)$. Let $(\rho_d, \rho_a) \vDash_I \phi_1$ and $\phi_1 \rightarrow (\psi_2[\vec{e}/\vec{a}] \vee \psi_2)$. Then,

$$(\rho_d, \rho_a) \vDash_I (\psi_2[\vec{e}/\vec{a}] \vee \psi_2) \tag{29}$$

Now given $\mathcal{T}_{dba}[\![Q_{ins}]\!](\rho_d, \rho_a) = (\rho_{d'}, \rho_{a'})$, we have two cases:

(i) Values of existing tuples will remain unchanged. Therefore,

$$(\rho_{d'},\ \rho_{a'}) \vDash_I \Psi_2\ when\ (\rho_d,\ \rho_a) \vDash_I \Psi_2 \tag{30}$$

(ii) For the newly inserted tuple, we have

$$(\rho_{d'},\ \rho_{a'}) \vDash_I \Psi_2\ when\ (\rho_d,\ \rho_a) \vDash_I \Psi_2[\vec{e}/\vec{a}] \tag{31}$$

Assuming $\Psi_2 \rightarrow \phi_2$, and combining Equations 29, 30, and 31, the lemma is proved for Q_{ins}.

- $\mathrm{Q}_{del} \triangleq \langle \vec{a} := \texttt{DELETE}(),\ cond \rangle$. Let $(\rho_d,\ \rho_a) \vDash_I \phi_1$. According to the definition of wp, $\mathsf{wp}(\langle \vec{a} := \texttt{DELETE}(),\ cond \rangle,\ \psi_2) = \psi_2 \wedge \neg cond$. Assume $(\rho_d,\ \rho_a) \vDash_I \phi_1$ and $\phi_1 \vDash_I (\psi_2 \wedge \neg cond)$. Therefore,

$$(\rho_d,\ \rho_a) \vDash (\psi_2 \wedge \neg cond) \tag{32}$$

Let $\mathscr{T}_{dba}[\![Q_{del}]\!](\rho_d,\ \rho_a)=(\rho_{d'},\ \rho_{a'})$. Since the tuples which remain intact in the database after the DELETE action, do not satisfy '$cond$', therefore, $(\rho_{d'},\ \rho_{a'}) \vDash_I \Psi_2$ when $(\rho_d,\ \rho_a) \vDash (\psi_2 \wedge \neg cond)$. Assuming $\Psi_2 \rightarrow \phi_2$, the lemma is proved for Q_{del}.

- $\mathrm{Q}_{sel} \triangleq \langle rs := \texttt{SELECT}(f(\vec{e}), r(\vec{h}(\vec{x})), \phi, g(\vec{e})),\ cond \rangle$. Since the SELECT statement does not affect any database attributes, this case can be proved easily as ϕ_1 and ϕ_2 only involve database attributes.

□