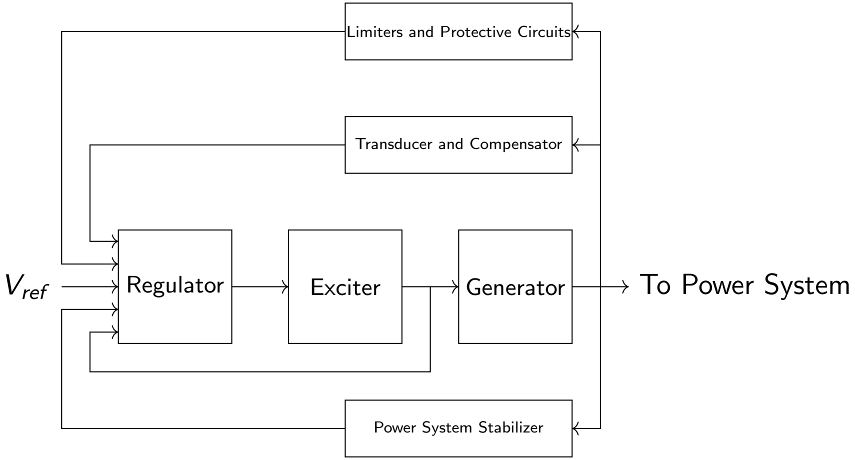
Excitation System

- It's basic function is to provide direct current to the synchronous machine field winding.
- It also performs control and protective functions essential to satisfactory performance of the power system by controlling the field voltage and thereby the field current.

Requirements :

1. Meet specified response criteria
2. Provide limiting and protective functions as required to prevent damage to itself, the generator and other equipment.
3. Meet specified requirements for operating flexibility.
4. Meet the desired reliability and availability.

Elements of an Excitation System



1. Exciter

   It provides dc power to the field winding.

2. Regulator

   It processes and amplifies input control signals to a level and form appropriate for control of the exciter.

3. Transducer and Compensator

   - It senses terminal voltage, rectifies and filters it to DC quantity and compares it with a reference.
   - It also provides load compensation.

4. Power System Stabilizer

   It provides an additional input signal to the regulator to damp power system oscillations. The input signal may be speed deviation, frequency deviation and accelerating power.

5. Limiters and Protective Circuits

   They ensure that the capability limits of the exciter and synchronous generator are not exceeded. Some of the functions are filed current limiter, maximum excitation limiter, under excitation limiter and terminal voltage limiter.

Types of Exciter

Exciter can be classified as follows:

1. DC Exciter
2. AC Exciter
3. Static Exciter

DC Exciter

- A self excited or separately excited DC generator is used.
- The DC generator is driven by a motor connected to the same shaft of the main synchronous generator.
- In case of separately excited DC generator, the field winding of the DC generator is excited by a three phase synchronous generator and three phase rectifier.
- The main field winding is connected to the DC generator through brushes and slip rings.
- This is called IEEE DC1A exciter.

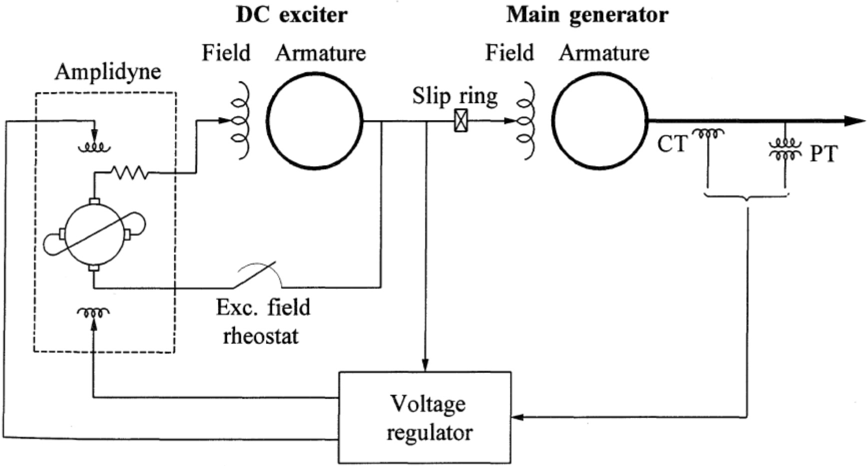


Figure: DC-Exciter

Source : P. Kundur

AC Exciter

- Synchronous generator whose armature is rotating and mounted on the same shaft is used.
- The output is rectified by a rotating rectifier and fed to the main filed winding.
- This does not require brushes and slip rings. Therefore, it is called a brush less excitation system.
- The field winding of the AC exciter generator is energised through a pilot permanent magnet AC generator whose $3-\phi$ output is converted to DC.
- This is called IEEE AC1A exciter.

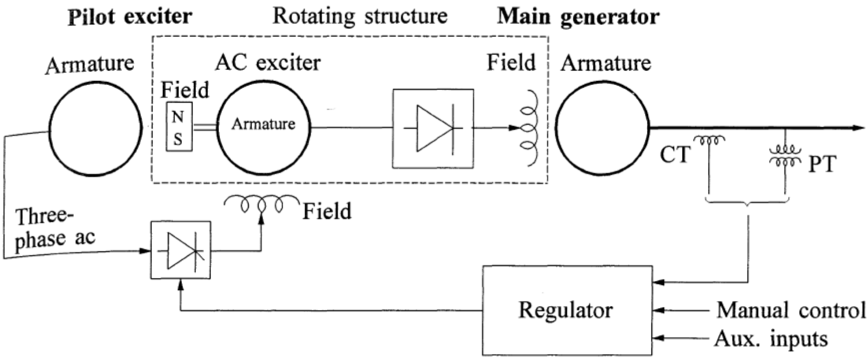


Figure: AC-Exciter

Source : P. Kundur

Static Exciter

1. In this exciter, the output of the main synchronous generator is converted from AC to DC through static rectification.
2. Then the output is supplied to the main generator field winding through slip rings.
3. It is called IEEE ST1A Exciter.

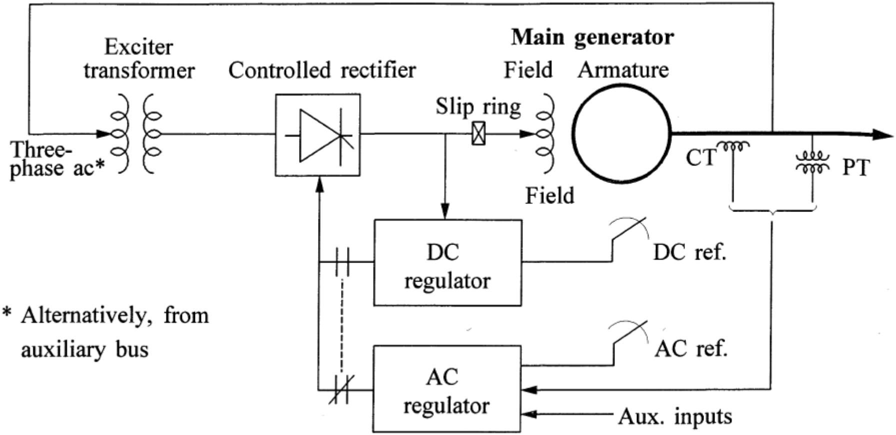


Figure: Static-Exciter

Source : P. Kundur

Modelling of Exciter

- Modelling of IEEE DC1A is done here.
- The same concept can be used to model AC and static exciter.

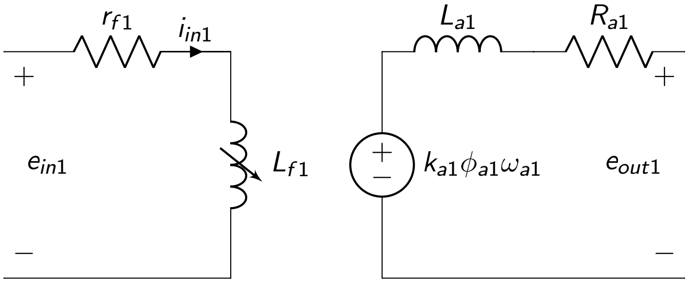


Figure: Separately excited DC exciter

$$v_{fd} = e_{out1}$$

$$e_{in1} = r_{f1} i_{in1} + L_{f1} \frac{di_{in1}}{dt} = r_{f1} i_{in1} + N_{f1} \frac{d\phi_{f1}}{dt} \tag{1}$$

$\phi_{a1} = \frac{\phi_{f1}}{\sigma}$. It means a fraction of ϕ_{f1} links the armature winding. The emf induced in the armature winding is

$$e_{a1} = k_{a1} \phi_{a1} \omega_{a1} \tag{2}$$

If R_a and L_a are neglected,

$$v_{fd} = e_{out1} = e_{a1} = k_{a1} \phi_{a1} \omega_{a1} \tag{3}$$

We can express ϕ_{f1} as

$$\phi_{f1} = \sigma \phi_{a1} = \frac{\sigma}{k_{a1} \omega_{a1}} v_{fd} \tag{4}$$

The field winding flux linkage is

$$\lambda_{f1} = L_{f1} i_{in1} = N_{f1} \phi_{f1} = \frac{\sigma N_{f1}}{k_{a1} \omega_{a1}} v_{fd} \tag{5}$$

$$\frac{v_{fd}}{i_{in1}} = \frac{k_{a1} \omega_{a1}}{\sigma N_{f1}} L_{f1} = k_g \tag{6}$$

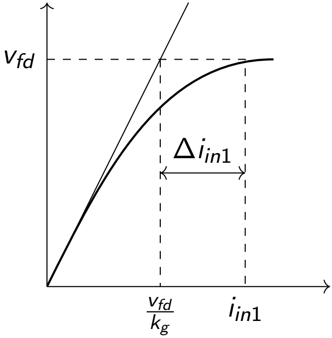


Figure: Excitation Saturation Characteristics

If saturation is neglected,

$$i_{in1} = \frac{v_{fd}}{k_g} \tag{7}$$

where k_g is the slope of the air gap line.
If saturation is considered,

$$i_{in1} = \frac{v_{fd}}{k_g} + \Delta i_{in1} = \frac{v_{fd}}{k_g} + f(v_{fd})v_{fd} \tag{8}$$

where $f(v_{fd})$ is the function representing the effect of saturation.
We can write eq (1) with the help of (8) and (4) as

$$e_{in1} = r_{f1}(\frac{v_{fd}}{k_g} + f(v_{fd})v_{fd}) + \frac{N_{f1}\sigma}{k_{a1}\omega_{a1}}\frac{dv_{fd}}{dt} \tag{9}$$

By using (6),

$$e_{in1} = r_{f1}(\frac{v_{fd}}{k_g} + f(v_{fd})v_{fd}) + \frac{L_{f1}}{k_g}\frac{dv_{fd}}{dt} \tag{10}$$

Let us divide (10) by $V_{fd,base}$ and multiply by $\frac{X_{md}}{R_{fd}}$.

$$\frac{X_{md}}{R_{fd}V_{fd,base}}e_{in1} = \frac{r_{f1}}{k_g}\frac{X_{md}}{R_{fd}V_{fd,base}}v_{fd} + r_{f1}f(v_{fd})\frac{X_{md}}{R_{fd}V_{fd,base}}v_{fd} + \frac{L_{f1}}{k_g}\frac{X_{md}}{R_{fd}V_{fd,base}}\frac{dv_{fd}}{dt} \tag{11}$$

We know that $V_{fd} = \frac{v_{fd}}{V_{fd,base}}$ and $E_{fd} = \frac{X_{md}}{R_{fd}}V_{fd}$.

$$V_R = K_E E_{fd} + r_{f1}f(\frac{R_{fd}}{X_{md}}E_{fd}V_{fd,base})E_{fd} + T_E\frac{dE_{fd}}{dt} \tag{12}$$

where

$$V_R = \frac{X_{md}}{R_{fd}V_{fd,base}}e_{in1}$$

$$K_E = \frac{r_{f1}}{k_g}$$

$$T_E = \frac{L_{f1}}{k_g}$$

$$S_E(E_{fd}) = r_{f1} f(\frac{R_{fd}}{X_{md}} E_{fd} V_{fd,base})$$

$$T_E \frac{dE_{fd}}{dt} = -(K_E + S_E(E_{fd}))E_{fd} + V_R \tag{13}$$

where $S_E(E_{fd})$ is a saturation function which is non linear. It can be approximated as

$$S_E(E_{fd}) = Ae^{BE_{fd}} \tag{14}$$

When evaluated at two points,

$$S_{Emax} = Ae^{BE_{fdmax}} \tag{15}$$

$$S_{E0.75max} = Ae^{B0.75E_{fdmax}} \tag{16}$$

For given values of K_E, V_{Rmax}, S_{Emax} and $S_{E0.75max}$, the constants A and B can be computed.

The voltage regulator is modelled as

$$T_A \frac{dV_R}{dt} = -V_R + K_A V_{in} \quad V_R^{\min} < V_R < V_R^{max} \tag{17}$$

where

$$T_A = \text{time constant}$$
$$K_A = \text{gain}$$

The transformer feeds back E_{fd} to the input. The output V_F is subtracted from V_R.

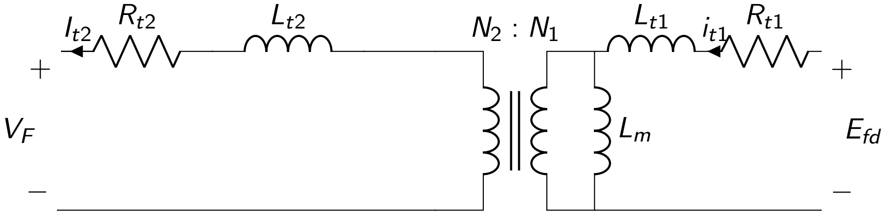


Figure: Stabilizing Transformer

Since the transformer secondary is connected to a large impedance circuit, $I_{t2} = 0$.

$$E_{fd} = R_{t1} i_{t1} + (L_{t1} + L_m)\frac{di_{t1}}{dt} \tag{18}$$

Since $i_{t2} = 0$,

$$V_F = \frac{N_2}{N_1} L_m \frac{di_{t1}}{dt} \tag{19}$$

Differentiating it once.

$$\frac{dV_F}{dt} = \frac{N_2}{N_1} L_m \frac{d^2 i_{t1}}{dt^2} \tag{20}$$

By differentiating (18) and rearranging,

$$\frac{d^2 i_{t1}}{dt^2} = \frac{1}{L_{t1} + L_m}\left(\frac{dE_{fd}}{dt} - \frac{R_{t1}}{L_m}\frac{N_1}{N_2} V_F\right) \tag{21}$$

From (20) and (21),

$$\frac{dV_F}{dt} = \frac{N_2}{N_1}\frac{L_m}{L_{t1} + L_m}\left(\frac{dE_{fd}}{dt} - \frac{R_{t1}}{L_m}\frac{N_1}{N_2} V_F\right) \tag{22}$$

Let

$$T_F = \frac{L_{t1} + L_m}{R_{t1}}, \quad K_F = \frac{N_2}{N_1}\frac{L_m}{R_{t1}}$$

Equation (22) can be written as

$$\frac{dV_F}{dt} = -\frac{1}{T_F}V_F + \frac{K_F}{T_F}\left(\frac{dE_{fd}}{dt}\right) \tag{23}$$

Let us define the rate feed back R_F.

$$R_F = \frac{K_F}{T_F}E_{fd} - V_F \tag{24}$$

Differentiating it once,

$$\frac{dR_F}{dt} = \frac{K_F}{T_F}\frac{dE_{fd}}{dt} - \frac{dV_F}{dt} \tag{25}$$

From (23) and (25),

$$\frac{dR_F}{dt} = \frac{1}{T_F} V_F$$
$$T_F \frac{dR_F}{dt} = V_F \tag{26}$$

From (22),

$$T_F \frac{dR_F}{dt} = -R_F + \frac{K_F}{T_F} E_{fd} \tag{27}$$

Finally, the DC1A exciter is represented as follows:

$$T_E \frac{dE_{fd}}{dt} = -(K_E + S_E(E_{fd}))E_{fd} + V_R \tag{28}$$
$$T_A \frac{dV_R}{dt} = -V_R + K_A R_F - \frac{K_A K_F}{T_F} E_{fd} + K_A(V_{ref} - V_t), \; V_R^{min} < V_R < V_R^{ma} \tag{29}$$
$$T_F \frac{dR_F}{dt} = -R_F + \frac{K_F}{T_F} E_{fd} \tag{30}$$

Modelling of Turbine

- Turbines provide mechanical input to the synchronous generators.
- Turbines can be of two types.
  1. Impulse Type - Example : Pelton Wheel
  2. Reaction Type - Example : Francis Turbine
- Turbines give rotational motion.
- Turbines are called prime movers.

The following turbines are modelled in this course.

1. Hydro Turbine
2. Steam Turbine

Hydro Turbine

Let

$$H = \text{Head i.e., the height from the gate to the reservoir}$$
$$G = \text{Gate position}$$
$$U = \text{Velocity}$$

The velocity of water is expressed as

$$U = K_u G \sqrt{H} \tag{31}$$

where K_u is a constant of proportionality.

If there is a small change in velocity ΔU from its initial condition $U_0, G_0 \& H_0$ equation (29) can be linearized as follows:

$$\Delta U = \left.\frac{\partial U}{\partial G}\right|_{G_0,H_0} \Delta G + \left.\frac{\partial U}{\partial H}\right|_{G_0,H_0} \Delta H \tag{32}$$

$$\Delta U = K_u \sqrt{H_0} \Delta G + \frac{K_u G_0}{2\sqrt{H_0}} \Delta H \tag{33}$$

$$U_0 = K_u G_0 \sqrt{H_0} \tag{34}$$

Equation (33) can be normalized as

$$\frac{\Delta U}{U_0} = \frac{1}{G_0} \Delta G + \frac{1}{2H_0} \Delta H \tag{35}$$

$$\Delta \bar{U} = \Delta \bar{G} + \frac{1}{2} \Delta \bar{H} \tag{36}$$

The mechanical input is given by

$$P_m = K_p H U \quad (37)$$

where K_p is a proportionality constant. This can also be normalized after linearizing around its initial condition.

$$\Delta \bar{P_m} = \Delta \bar{H} + \Delta \bar{U} \quad (38)$$

Substituting (36) in (38),

$$\Delta \bar{P_m} = 1.5 \Delta \bar{H} + \Delta \bar{G} \quad (39)$$

From the Newton's second law, the acceleration due to the change in head can be expressed as

$$\rho LA\frac{dU}{dt} = -A\rho a_g \Delta H \tag{40}$$

where

$$\begin{aligned}
\rho &= \text{ water density in } kg/m^3 \\
L &= \text{Length of the conduit in m} \\
A &= \text{Area of the conduit in } m^2 \\
a_g &= \text{acceleration due to gravity in } m/s^2 \\
U &= \text{velocity in } m/s \\
H &= \text{Head in m}
\end{aligned}$$

To normalize,

$$\frac{\rho LA\frac{dU}{dt}}{A\rho a_g U_0 H_0} = -\frac{A\rho a_g \Delta H}{A\rho a_g U_0 H_0} \tag{41}$$

After rearranging,

$$\frac{LU_0}{a_g H_0}\frac{d\bar{U}}{dt} = -\Delta \bar{H} \tag{42}$$

$$T_w \frac{d\bar{U}}{dt} = -\Delta \bar{H} \tag{43}$$

where $T_w = \frac{LU_0}{a_g H_0}$ in seconds. It is called as water starting time and it the time required for a head H_0 to accelerate the water to a velocity U_0. Equation (43) can be written using the Laplace transform.

$$T_w s \Delta \bar{U}(s) = -\Delta \bar{H}(s) \tag{44}$$

Substituting (36) in (43) and taking the Laplace transform,

$$T_w s \Delta \bar{U}(s) = 2(\Delta \bar{G}(s) - \Delta \bar{U}(s)) \tag{45}$$

$$\Delta \bar{U}(s) = \frac{1}{1 + \frac{1}{2} s T_w} \Delta \bar{G}(s) \tag{46}$$

From (39) and (36),

$$\Delta \bar{P_m}(s) = 3\Delta \bar{U}(s) - 2\Delta \bar{G}(s) \tag{47}$$

Substituting (46) in (47),

$$\Delta \bar{P_m}(s) = \frac{1 - T_w s}{1 + \frac{1}{2} s T_w} \Delta \bar{G}(s) \tag{48}$$

Since the above model is linear, it is only valid for small changes. Equation (48) can be represented in time domain,

$$T_w \frac{d\Delta P_m}{dt} = 2\left(-\Delta P_m + \Delta G - \frac{d\Delta G}{dt}\right) \tag{49}$$

The actual parameters are

$$\begin{aligned} P_m &= P_{m0} + \Delta P_m \\ G &= G_0 + \Delta G \end{aligned} \tag{50}$$

Equation (49) can be written as

$$T_w \frac{dP_m}{dt} = 2\left(-P_m + G - \frac{dG}{dt}\right) \tag{51}$$

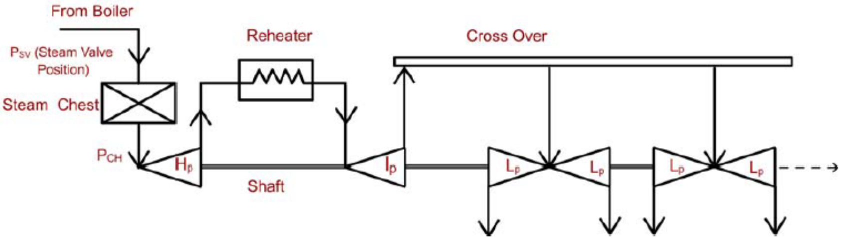
To convert it into per unit form, let us divide by S_{base}.

$$\boxed{T_w \frac{dT_m}{dt} = 2\left(-T_m + G_{pu} - \frac{dG_{pu}}{dt}\right)} \tag{52}$$

In per unit form , $P = T$. T_m is the per unit mechanical output torque of a turbine.

Equation (52) describes the dynamic behavior of a hydro turbine.

Steam Turbine



- Steam plants consist of a fuel supply to a steam boiler that supplies a steam chest.
- The steam chest contains pressurized steam that enters a high pressure (HP) turbine through a steam valve.
- It is common to include additional stages, such as the intermediate (IP) and low (LP) pressure turbines.
- In this model, we are interested in the effect of the steam valve position (power P_{SV}) on the synchronous machine torque T_m.

The incremental steam chest dynamic model is a simple linear single time constant with unity gain.

$$T_{CH}\frac{d\Delta P_{CH}}{dt} = -\Delta P_{CH} + \Delta P_{SV} \tag{53}$$

$$\begin{aligned} T_{CH} &= \text{time delay of the steam chest} \\ \Delta P_{CH} &= \text{change in output power of the steam chest} \\ \Delta P_{SV} &= \text{change in the steam valve position} \end{aligned}$$

Let the fraction of ΔP_{CH} be converted to Torque.

$$\Delta T_{HP} = K_{HP}\Delta P_{CH} \tag{54}$$

The remaining fraction $(1 - K_{HP})\Delta P_{CH}$ enters the reheater.

The reheat process has a time delay that can be modeled similarly as

$$T_{RH}\frac{d\Delta P_{RH}}{dt} = -\Delta P_{RH} + (1 - K_{HP})\Delta P_{CH} \tag{55}$$

where ΔP_{RH} is the change in is the change in output power of the reheater. Assuming that this output is totally converted into torque on the LP turbine,

$$\Delta T_{LP} = \Delta P_{RH} \tag{56}$$

The total torque is

$$\Delta T_M = \Delta T_{HP} + \Delta T_{LP} = K_{HP}\Delta P_{CH} + \Delta P_{RH} \tag{57}$$

Substituting (57) in (55) and using (53), we get,

$$T_{RH}\frac{d\Delta T_M}{dt} = -\Delta T_M + \left(1 - \frac{K_{HP}T_{RH}}{T_{CH}}\right)\Delta P_{CH} + \frac{K_{HP}T_{RH}}{T_{CH}}\Delta P_{SV} \tag{58}$$

The actual variables are

$$T_M = T_{M0} + \Delta T_M; \quad P_{CH} = P_{CH0} + \Delta P_{CH}; \quad P_{SV} = P_{SV0} + \Delta P_{SV}$$

where T_{M0}, P_{CH0} and P_{SV0} are the initial operating conditions.
The steam turbine model is as follows:

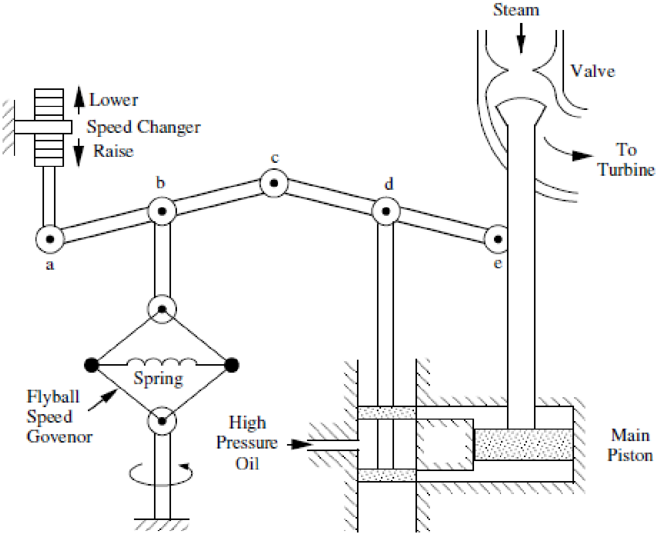
$$T_{RH}\frac{dT_M}{dt} = -\Delta T_M + \left(1 - \frac{K_{HP}T_{RH}}{T_{CH}}\right)P_{CH} + \frac{K_{HP}T_{RH}}{T_{CH}}P_{SV} \tag{59}$$

$$T_{CH}\frac{dP_{CH}}{dt} = -P_{CH} + P_{SV} \tag{60}$$

For a non reheat system, $T_{RH} = 0$ and the following model is used.
$P_{CH} = T_M$ since $K_{HP} = 1$.

$$\boxed{T_{CH}\frac{dP_{CH}}{dt} = -T_M + P_{SV}} \tag{61}$$

Speed Governor



To model this action, we analyze the linkages and note that any incremental change in the positions of points a, b, and c are related by

$$\Delta y_b = K_{ba}\Delta y_a + K_{bc}\Delta y_c \tag{62}$$

Any incremental change in the position of points c, d, and e are related by

$$\Delta y_d = K_{dc}\Delta y_c + K_{de}\Delta y_e \tag{63}$$

The point a is related to the change in the reference power and if the reference power is P_{ref},

$$\Delta P_{ref} = K_a\Delta y_a \tag{64}$$

The point b changes according to the change in the speed of the shaft and is proportional to it and hence

$$\Delta\omega = \omega_{base} K_b\Delta y_b \tag{65}$$

A change in the position of point d affects the position of point e through the time delay associated with the liquid in the servo. We assume a linear dynamic response for this time delay:

$$\frac{d\Delta y_e}{dt} = -K_e \Delta y_d \tag{66}$$

Substituting (63) in (66), we get

$$\frac{d\Delta y_e}{dt} = -K_e K_{dc} \Delta y_c - K_e K_{de} \Delta y_e \tag{67}$$

From (62), we can get Δy_c. The same is substituted in (67).

$$\begin{aligned} \frac{d\Delta y_e}{dt} &= -K_e K_{dc} \left(\frac{\Delta y_b - K_{ba} \Delta y_a}{K_{bc}} \right) - K_e K_{de} \Delta y_e \\ &= -\frac{K_e K_{dc}}{K_{bc}} \Delta y_b + \frac{K_e K_{dc} K_{ba}}{K_{bc}} \Delta y_a - K_e K_{de} \Delta y_e \end{aligned} \tag{68}$$

We can substitute Δy_a and Δy_b from (64) and (65) in (68).

$$\frac{d\Delta y_e}{dt} = -\frac{K_e K_{dc}}{K_{bc}} \frac{\Delta\omega}{\omega_{base} K_b} + \frac{K_e K_{dc} K_{ba}}{K_{bc}} \frac{\Delta P_{ref}}{K_a} - K_e K_{de} \Delta y_e \quad (69)$$

Let us multiply (69) by $\frac{K_a K_{bc}}{K_e K_{dc} K_{ba}}$.

$$\frac{K_a K_{bc}}{K_e K_{dc} K_{ba}} \frac{d\Delta y_e}{dt} = -\frac{K_a}{K_b K_{ba}} \frac{\Delta\omega}{\omega_{base}} + \Delta P_{ref} - \frac{K_a K_{bc} K_{de}}{K_{ba} K_{dc}} \Delta y_e \quad (70)$$

Let us multiply and divide the LHS by K_{de}.

$$\frac{1}{K_e K_{de}} \frac{K_a K_{de} K_{bc}}{K_{dc} K_{ba}} \frac{d\Delta y_e}{dt} = -\frac{K_a}{K_b K_{ba}} \frac{\Delta\omega}{\omega_{base}} + \Delta P_{ref} - \frac{K_a K_{bc} K_{de}}{K_{ba} K_{dc}} \Delta y_e \quad (71)$$

Using the proportionality between ΔP_{SV} and Δy_e,

$$\Delta P_{SV} = \frac{K_a K_{bc} K_{de}}{K_{ba} K_{dc}} \Delta y_e \tag{72}$$

and defining the following

$$R_D = \frac{K_b K_{ba}}{K_a} \tag{73}$$

$$T_{SV} = \frac{1}{K_e K_{de}} \tag{74}$$

$$T_{SV} \frac{d\Delta P_{SV}}{dt} = -\frac{1}{R_D} \frac{\Delta \omega}{\omega_{base}} + \Delta P_{ref} - \Delta P_{SV} \tag{75}$$

The actual variables are

$$P_{SV} = P_{SV0} + \Delta P_{SV}; \quad P_{ref} = P_{ref0} + \Delta P_{ref}; \quad \omega = \omega_{base} + \Delta\omega$$

The speed governor model is

$$\boxed{T_{SV}\frac{dP_{SV}}{dt} = -\frac{1}{R_D}\left(\frac{\omega}{\omega_{base}} - 1\right) + P_{ref} - P_{SV}, \quad 0 \leq P_{SV} \leq P_{SV}^{max}} \tag{76}$$

where R_D is the speed regulation. It depends on the droop, the amount by which the frequency falls from no load to full load without change in the input power.

$$R_D = \frac{2\pi \text{ droop}}{\omega_{base}} \tag{77}$$

where "droop" is expressed in Hz/ per unit MW. If droop is expressed in percentage,

$$R_D = \frac{\% \text{ droop}}{100} \tag{78}$$

Load Modelling

Loads can be categorized as

1. Static Loads
    - Heating Loads
    - Lighting Loads
2. Dynamic Loads
    - Synchronous Motors
    - Induction Motors

Static Loads

Static loads can be represented as

1. constant power
2. constant current
3. constant impedance

- Voltage and frequency affect static loads.

The effect of voltage change on static loads is represented as

$$P = P_0 \left(\frac{V}{V_0} \right)^{np} \tag{79}$$

$$Q = Q_0 \left(\frac{V}{V_0} \right)^{nq} \tag{80}$$

$$P_0, Q_0 = \text{Real and reactive power at } V_0$$

$$P, Q = \text{Real and reactive power at } V$$

Static Loads (contd.)

When

$$np, nq = 0 \text{(Constant power load)}$$
$$np, nq = 1 \text{(Constant current load)}$$
$$np, nq = 2 \text{(Constant impedance load)}$$

For aggregate loads,

$$np = 0.5 \ to \ 1.8$$
$$nq = 1.5 \ to \ 6$$

Static Loads (contd.)

A polynomial to represent all types is given below.

$$P = P_0 \left(P_Z \left(\frac{V}{V_0} \right)^2 + P_I \left(\frac{V}{V_0} \right) + P_P \right) \tag{81}$$

$$Q = Q_0 \left(Q_Z \left(\frac{V}{V_0} \right)^2 + Q_I \left(\frac{V}{V_0} \right) + Q_P \right) \tag{82}$$

(83)

It is called ZIP load. $P_Z, P_I, P_P, Q_Z, Q_I, Q_P$ are constants representing the approximated fraction of the aggregated load taken as constant impedance, current and power loads, respectively.

- Constant impedance loads are real. (Heating and Lighting loads)
- Constant real and power loads are not real but used to represent dynamic loads as static loads after a disturbance.

Static Loads (contd.)

Hence it is necessary to take the effect of frequency variation also on the loads to improve the accuracy and can be represented as

$$P = P_0 \left(P_Z \left(\frac{V}{V_0} \right)^2 + P_I \left(\frac{V}{V_0} \right) + P_P \right) (1 + K_{pf} \Delta f) \tag{84}$$

$$Q = Q_0 \left(Q_Z \left(\frac{V}{V_0} \right)^2 + Q_I \left(\frac{V}{V_0} \right) + Q_P \right) (1 + K_{qf} \Delta f) \tag{85}$$

where

$$\begin{aligned} \Delta f &= f - f_0 \\ K_{pf} &= 0 \ to \ 3 \\ K_{qf} &= -2 \ to \ 6 \end{aligned}$$

Static loads are mostly represented as constant Z type for computational simplicity.

The dynamic equations of a synchronous generator can be used to represent the dynamics of motors with a modification in the following equation.

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = T_e + D(\omega - \omega_{base}) - T_m \tag{86}$$

In motors,

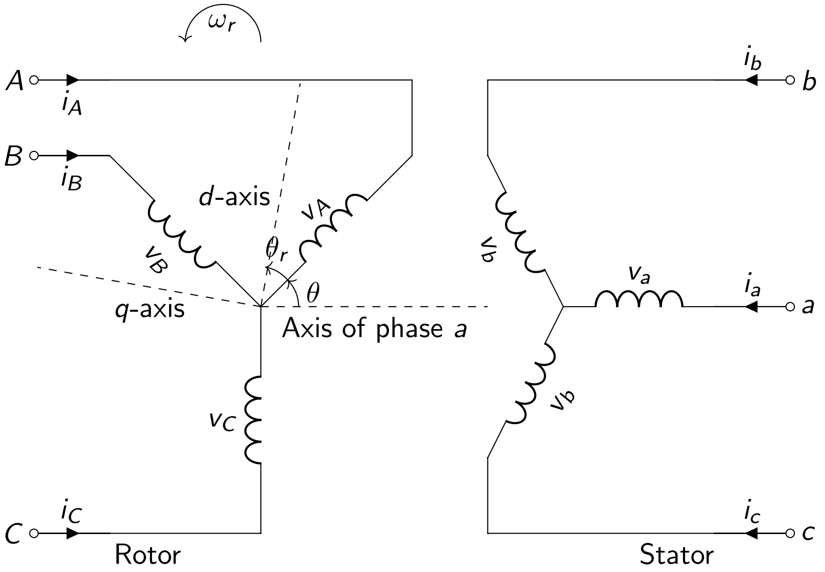
- T_e is the input.
- T_m is the output.
- Currents entering the stator windings are taken positive.

Induction Motor

- The stator of an induction machine is the same as a synchronous machine. It has three phase windings placed 120° electrical apart in space.
- There are two types of rotor. One is squirrel cage and the other one is phase-wound.
- The phase wound rotor has three phase windings placed 120° electrical apart in space.
- The squirrel cage rotor also behaves as a short circuited three phase winding.

The following points are to be noted while modelling an induction machine.

- The rotor has a symmetrical structure. Therefore, d-axis and q-axis circuits are identical.
- The rotor speed is not fixed but varies with load. This has an impact on the selection of $d-q$ reference frame.
- There is no excitation source applied to the rotor windings. Hence, the dynamics of the rotor circuits are determined by the slip.
- The currents induced in the shorted rotor windings produce a revolving filed with the same number of poles and speed as that of the filed produced by the stator windings. Therefore, rotor windings may be modelled by an equivalent three phase winding.


ω_r
A
i_A
B
i_B
C
i_C
d-axis
q-axis
v_A
v_B
v_C
θ_r
θ
Axis of phase a
Rotor
i_b
b
i_a
a
i_c
c
v_b
v_a
v_b
Stator

With a constant rotor angular velocity of ω_r in electrical rad/sec,

$$\theta = \omega_r t \tag{87}$$

With a slip s,

$$\theta = (1 - s)\omega_s t \tag{88}$$

where ω_s is the synchronous speed electrical rad/sec. Let us assume that $d - q$ axis is rotating at a speed of ω_s. The axis of rotor phase-A has an angle of θ_r.

$$\theta_r = \omega_s t - \theta = s\omega_s t \tag{89}$$

$$\frac{d\theta_r}{dt} = s\omega_s \tag{90}$$

The stator and rotor circuits voltage equations are as follows:

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} r_s & 0 & 0 \\ 0 & r_s & 0 \\ 0 & 0 & r_s \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \end{bmatrix} \tag{91}$$

$$\begin{bmatrix} v_A \\ v_B \\ v_C \end{bmatrix} = \begin{bmatrix} r_r & 0 & 0 \\ 0 & r_r & 0 \\ 0 & 0 & r_r \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda_A \\ \lambda_B \\ \lambda_C \end{bmatrix} \tag{92}$$

- The self inductances of stator windings are assumed to be equal (l_{aa})
- The mutual inductances between stator windings are the same (l_{ab}).
- The maximum mutual inductance between stator and rotor windings is l_{aA}.

The flux linkages of stator and rotor can be written as

$$\begin{bmatrix}\lambda_a\\ \lambda_b\\ \lambda_c\end{bmatrix} = \begin{bmatrix} l_{aa} & l_{ab} & l_{ab}\\ l_{ab} & l_{aa} & l_{ab}\\ l_{ab} & l_{ab} & l_{aa}\end{bmatrix}\begin{bmatrix} i_a\\ i_b\\ i_c\end{bmatrix} + \begin{bmatrix} l_{aA}\cos\theta & l_{aA}\cos(\theta-120^\circ) & l_{aA}\cos(\theta+120^\circ)\\ l_{aA}\cos(\theta-120^\circ) & l_{aA}\cos\theta & l_{aA}\cos(\theta+120^\circ)\\ l_{aA}\cos(\theta+120^\circ) & l_{aA}\cos(\theta-120^\circ) & l_{aA}\cos\theta\end{bmatrix} \tag{93}$$

$$\begin{bmatrix}\lambda_A\\ \lambda_B\\ \lambda_C\end{bmatrix} = \begin{bmatrix} l_{aA}\cos\theta & l_{aA}\cos(\theta-120^\circ) & l_{aA}\cos(\theta+120^\circ)\\ l_{aA}\cos(\theta-120^\circ) & l_{aA}\cos\theta & l_{aA}\cos(\theta+120^\circ)\\ l_{aA}\cos(\theta+120^\circ) & l_{aA}\cos(\theta-120^\circ) & l_{aA}\cos\theta\end{bmatrix}\begin{bmatrix} i_a\\ i_b\\ i_c\end{bmatrix} + \begin{bmatrix} l_{AA} & l_{AA} & l_{AB}\\ l_{AB} & l_{AA} & l_{AB}\\ l_{AB} & l_{AB} & l_{AA}\end{bmatrix} \tag{94}$$

- For the stator winding, the same $dq0$ transformation used in synchronous machines can be used.
- Since the rotor is not rotating at ω_s, a different $dq0$ transformation matrix is required .

$$T_{dq0}^{s} = \frac{2}{3}\begin{bmatrix} \cos\omega_s t & \cos(\omega_s t - 120^\circ) & \cos(\omega_s t + 120^\circ) \\ -\sin\omega_s t & -\sin(\omega_s t - 120^\circ) & -\sin(\omega_s t + 120^\circ) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{95}$$

$$T_{dq0}^{r} = \frac{2}{3}\begin{bmatrix} \cos((\omega_s - \omega_r)t) & \cos((\omega_s - \omega_r)t - 120^\circ) & \cos((\omega_s - \omega_r)t + 120^\circ) \\ -\sin((\omega_s - \omega_r)t) & -\sin((\omega_s - \omega_r)t - 120^\circ) & -\sin((\omega_s - \omega_r)t + 120^\circ) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{96}$$

Applying the transformation to flux linkage equations and assuming

$$l_{ss} = l_{aa} - l_{ab}$$
$$l_{rr} = l_{AA} - l_{AB}$$
$$l_m = \frac{3}{2} l_{aA}$$

We get,

$$\begin{bmatrix}\lambda_{ds}\\ \lambda_{qs}\\ \lambda_{0s}\end{bmatrix} = \begin{bmatrix} l_{ss} & 0 & 0\\ 0 & l_{ss} & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix} i_{ds}\\ i_{qs}\\ i_{0s}\end{bmatrix} + \begin{bmatrix} l_m & 0 & 0\\ 0 & l_m & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix} i_{dr}\\ i_{qr}\\ i_{0r}\end{bmatrix} \tag{97}$$

$$\begin{bmatrix}\lambda_{dr}\\ \lambda_{qr}\\ \lambda_{0r}\end{bmatrix} = \begin{bmatrix} l_m & 0 & 0\\ 0 & l_m & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix} i_{ds}\\ i_{qs}\\ i_{0s}\end{bmatrix} + \begin{bmatrix} l_{rr} & 0 & 0\\ 0 & l_{rr} & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix} i_{dr}\\ i_{qr}\\ i_{0r}\end{bmatrix} \tag{98}$$

Similarly, the voltage equations in $dq0$ reference frame are

$$\begin{bmatrix} v_{ds}\\ v_{qs}\\ v_{0s}\end{bmatrix} = \begin{bmatrix} r_s & 0 & 0\\ 0 & r_s & 0\\ 0 & 0 & r_s\end{bmatrix}\begin{bmatrix} i_{ds}\\ i_{qs}\\ i_{0s}\end{bmatrix} + \begin{bmatrix} 0 & -\omega_s & 0\\ \omega_s & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\lambda_{ds}\\ \lambda_{qs}\\ \lambda_{0s}\end{bmatrix} + \frac{d}{dt}\begin{bmatrix}\lambda_{ds}\\ \lambda_{qs}\\ \lambda_{0s}\end{bmatrix} \tag{99}$$

$$\begin{bmatrix} v_{dr}\\ v_{qr}\\ v_{0r}\end{bmatrix} = \begin{bmatrix} r_r & 0 & 0\\ 0 & r_r & 0\\ 0 & 0 & r_r\end{bmatrix}\begin{bmatrix} i_{dr}\\ i_{qr}\\ i_{0r}\end{bmatrix} + \begin{bmatrix} 0 & -(\omega_s - \omega_r) & 0\\ (\omega_s - \omega_r) & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\lambda_{dr}\\ \lambda_{qr}\\ \lambda_{0r}\end{bmatrix} + \frac{d}{dt}\begin{bmatrix}\lambda_{dr}\\ \lambda_{qr}\\ \lambda_{0r}\end{bmatrix} \tag{100}$$

The rotor input can be expressed as

$$
\begin{aligned}
S_{rotor} &= \begin{bmatrix} v_A \\ v_B \\ v_C \end{bmatrix}^T \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} = \begin{bmatrix} v_{dr} \\ v_{qr} \\ v_{0r} \end{bmatrix}^T ((T^r_{dq0})^T)^{-1}(T^r_{dq0})^{-1} \begin{bmatrix} i_{dr} \\ i_{qr} \\ i_{0r} \end{bmatrix} \\
&= \frac{3}{2}(r_r(i^2_{dr} + i^2_{qr})) + \frac{3}{2}(\lambda_{dr} i_{qr} - \lambda_{qr} i_{dr})(\omega_s - \omega_r) \\
&+ \left(i_{dr}\frac{d\lambda_{dr}}{dt} + i_{qr}\frac{d\lambda_{qr}}{dt} \right)
\end{aligned} \tag{101}
$$

The second component is the product of torque and the difference of speed. The rotor speed with respect to the dq-axis is $(\omega_r - \omega_s)$ electrical rad/sec. Therefore, T_e in N-m for a P pole machine is

$$
T_e = \frac{3}{2}\frac{P}{2}(\lambda_{qr} i_{dr} - \lambda_{dr} i_{qr}) \tag{102}
$$

Per Unit Representation

The same base quantities can be taken for both stator and rotor.
Let

$$V_{base} = \text{peak value of rated phase voltage, V}$$
$$I_{base} = \text{peak value of rated current, A}$$
$$f_{base} = \text{rated frequency, Hz}$$

$$Z_{base} = \frac{V_{base}}{I_{base}}\ \Omega,\ \omega_{base} = \omega_s = 2\pi f \text{ elect.rad/sec},\ L_{base} = \frac{Z_{base}}{\omega_{base}}\ \text{H},$$
$$\lambda_{base} = \frac{V_{base}}{\omega_{base}},\ S_{base} = \frac{3}{2} V_{base} I_{base}\ \text{MVA},\ T_{base} = \frac{3}{2}\frac{P}{2}\lambda_{base} I_{base}\ \text{N-m}$$

$$
\begin{aligned}
V_{ds} = \frac{v_{ds}}{V_{base}},\ V_{qs} = \frac{v_{qs}}{V_{base}},\ V_{dr} = \frac{v_{dr}}{V_{base}},\ V_{qr} = \frac{v_{qr}}{V_{base}} \\
I_{ds} = \frac{i_{ds}}{I_{base}},\ I_{qs} = \frac{i_{qs}}{I_{base}},\ I_{dr} = \frac{i_{dr}}{I_{base}},\ I_{qr} = \frac{i_{qr}}{I_{base}} \\
\psi_{ds} = \frac{\lambda_{ds}}{\lambda_{base}},\ \psi_{qs} = \frac{\lambda_{qs}}{\lambda_{base}},\ \psi_{dr} = \frac{\lambda_{dr}}{\lambda_{base}},\ \psi_{qr} = \frac{\lambda_{qr}}{\lambda_{base}} \\
X_s = \frac{\omega_{base} l_{ss}}{Z_{base}},\ X_r = \frac{\omega_{base} l_{rr}}{Z_{base}},\ X_m = \frac{\omega_{base} l_m}{Z_{base}},\ R_s = \frac{r_s}{Z_{base}} \\
R_r = \frac{r_r}{Z_{base}},\ T_e = \frac{T_e}{T_{base}},\ \omega_{r(p.u)} = \frac{\omega_r}{\omega_{base}} = (1-s)
\end{aligned}
\tag{103}
$$

The equations (99) - (102) can be written in per unit as given below.

$$V_{ds} = R_s I_{ds} - \psi_{qs} + \frac{1}{\omega_{base}} \frac{d\psi_{ds}}{dt} \tag{104}$$

$$V_{qs} = R_s I_{qs} + \psi_{ds} + \frac{1}{\omega_{base}} \frac{d\psi_{qs}}{dt} \tag{105}$$

$$V_{dr} = R_r I_{dr} - s\psi_{qr} + \frac{1}{\omega_{base}} \frac{d\psi_{dr}}{dt} \tag{106}$$

$$V_{qr} = R_r I_{qr} + s\psi_{dr} + \frac{1}{\omega_{base}} \frac{d\psi_{qr}}{dt} \tag{107}$$

$$\psi_{ds} = X_s I_{ds} + X_m I_{dr} \tag{108}$$

$$\psi_{qs} = X_s I_{qs} + X_m I_{qr} \tag{109}$$

$$\psi_{dr} = X_r I_{dr} + X_m I_{ds} \tag{110}$$

$$\psi_{qr} = X_r I_{qr} + X_m I_{qs} \tag{111}$$

The stator transients are neglected. Therefore, the equations (104) and (105) can be written as

$$V_{ds} = R_s I_{ds} - \psi_{qs} \tag{112}$$
$$V_{qs} = R_s I_{qs} + \psi_{ds} \tag{113}$$

Finding I_{dr} and I_{qr} from (110) and (111) and substituting in (108) and (109), we get

$$\psi_{ds} = X_s' I_{ds} + V_q' \tag{114}$$
$$\psi_{qs} = X_s' I_{qs} - V_d' \tag{115}$$

where

$$X_s' = \left(X_s - \frac{X_m^2}{X_r} \right), \; V_d' = -\frac{X_m}{X_r}\psi_{qr}, \; V_q' = \frac{X_m}{X_r}\psi_{dr}$$

For a squirrel cage rotor, $V_{dr} = V_{qr} = 0$. Finding I_{dr} and I_{qr} from (110) and (111), substituting in (106) and (107) and using (114) and (115), we get

$$T_0' \frac{dV_d'}{dt} = -(V_d' + (X_s - X_s')I_{qs}) + s\omega_{base} T_0' V_q' \tag{116}$$

$$T_0' \frac{dV_q'}{dt} = -(V_q' - (X_s - X_s')I_{ds}) - s\omega_{base} T_0' V_d' \tag{117}$$

$$\tag{118}$$

where

$$T_0' = \frac{X_r}{\omega_{base} R_r}$$

The per unit torque is

$$T_e = \frac{T_e}{T_{base}} = \frac{\frac{3}{2}\frac{P}{2}(\lambda_{qr} i_{dr} - \lambda_{dr} i_{qr})}{\frac{3}{2}\frac{P}{2}\lambda_{base} I_{base}} = (\psi_{qr} I_{dr} - \psi_{dr} I_{qr}) \tag{119}$$

The equation of motion is given by

$$J\frac{d\omega_m}{dt} = T_e - T_m \tag{120}$$

where

$$\omega_m = \text{speed of the rotor in mechanical rad/sec}$$
$$T_e = \text{electrical torque in N-m}$$
$$T_m = \text{mechanical torque in N-m}$$

Dividing (120) by $S_{base} = T_{base}\omega_{m,base}$,

$$\frac{J}{S_{base}}\frac{d\omega_m}{dt} = \frac{T_e - T_m}{T_{base}\omega_{m,base}}$$

$$\frac{J\omega_{m,base}}{S_{base}}\omega_{m,base}\frac{d\omega_m}{dt\omega_{m,base}} = \frac{T_e - T_m}{T_{base}} \quad (121)$$

$$2H\frac{d\omega_{r(p.u)}}{dt} = T_e - T_m$$

where

$$H = \frac{\frac{1}{2}J\omega_{m,base}^2}{S_{base}}$$

$$\omega_{r(p.u)} = \frac{\omega_r}{\omega_{base}} = \frac{\omega_m}{\omega_{base}}$$

Since $\omega_{r(p.u)} = (1 - s)$,

$$-2H\frac{ds}{dt} = T_e - T_m \quad (122)$$

Substituting (114) and (115) in (113) and (112), respectively,

$$V_{ds} = R_s I_{ds} - X_s' I_{qs} + V_d' \tag{123}$$

$$V_{qs} = R_s I_{qs} + X_s' I_{ds} + V_q' \tag{124}$$

Let the three phase stator voltages be

$$v_a = \sqrt{2} V_t \cos(\omega_s t + \alpha) \tag{125}$$

$$v_b = \sqrt{2} V_t \cos(\omega_s t + \alpha - 120^\circ) \tag{126}$$

$$v_c = \sqrt{2} V_t \cos(\omega_s t + \alpha + 120^\circ) \tag{127}$$

Applying the *dq*0 transformation and dividing by the base voltage,

$$\begin{bmatrix} V_{ds} \\ V_{qs} \end{bmatrix} = \begin{bmatrix} V_{t(p.u)} \cos\alpha \\ V_{t(p.u)} \sin\alpha \end{bmatrix} \tag{128}$$

where $V_{t(p.u)} = \frac{V_t}{V_{base}}$.

The dq component of staor voltages can be represented as a phasor as follows:

$$V_{ds} + \jmath V_{qs} = V_{t(p.u)} e^{\jmath \alpha} \tag{129}$$

Similarly,

$$I_{ds} + \jmath I_{qs} = I_{t(p.u)} e^{\jmath \gamma} \tag{130}$$

$(123) + \jmath(124)$,

$$\begin{aligned} V_{t(p.u)} e^{\jmath \alpha} &= V_{ds} + \jmath V_{qs} = (R_s + \jmath X'_s)(I_{ds} + \jmath I_{qs}) + (V'_d + \jmath V'_q) \\ V_{t(p.u)} e^{\jmath \alpha} &= (R_s + \jmath X'_s) I_{t(p.u)} e^{\jmath \gamma} + (V'_d + \jmath V'_q) \end{aligned} \tag{131}$$

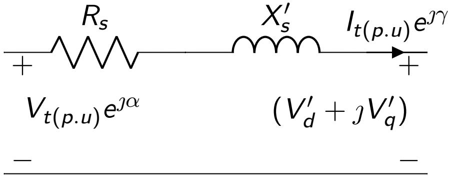


Figure: Electrical equivalent circuit of an induction motor

The complete induction machine model is

$$\frac{1}{\omega_{base}}\frac{d\theta_r}{dt} = \frac{\omega_{base} - \omega_r}{\omega_{base}} = s \quad (132)$$

$$2H\frac{ds}{dt} = -(T_e - T_m) \quad (133)$$

$$T_0'\frac{dV_d'}{dt} = -(V_d' + (X_s - X_s')I_{qs}) + s\omega_{base}T_0'V_q' \quad (134)$$

$$T_0'\frac{dV_q'}{dt} = -(V_q' - (X_s - X_s')I_{ds}) - s\omega_{base}T_0'V_d' \quad (135)$$

$$T_e = (\psi_{qr}I_{dr} - \psi_{dr}I_{qr}) = (V_d'I_{ds} + V_q'I_{qs}) \quad (136)$$

$$V_{t(p.u)}e^{\jmath\alpha} = (R_s + X_s')I_{t(p.u)}e^{\jmath\gamma} + (V_d' + \jmath V_q') \quad (137)$$

Initial Conditions

From (134) and (135),

$$(V_d' + (X_s - X_s')I_{qs}) - s\omega_{base} T_0' V_q' = 0 \tag{138}$$
$$(V_q' - (X_s - X_s')I_{ds}) + s\omega_{base} T_0' V_d' = 0 \tag{139}$$

$(138) + \jmath(139)$,

$$V_d' + \jmath V_q' - \jmath(X_s - X_s')(I_{ds} + \jmath I_{qs}) + \jmath\omega_{base} s T_0'(V_d' + \jmath V_q') = 0 \tag{140}$$

$$\frac{V_d' + \jmath V_q'}{(I_{ds} + \jmath I_{qs})} = \frac{\jmath(X_s - X_s')}{(1 + \jmath\omega_{base} s T_0')}$$
$$V_d' + \jmath V_q' = (I_{ds} + \jmath I_{qs})\frac{\jmath(X_s - X_s')}{(1 + \jmath\omega_{base} s T_0')} = I_{t(p.u)} e^{\jmath\gamma} \frac{\jmath(X_s - X_s')}{(1 + \jmath\omega_{base} s T_0')} \tag{141}$$

Substituting (141) in (131),

$$V_{t(p.u)}e^{\jmath\alpha} = \left((R_s + \jmath X_s') + \frac{\jmath(X_s - X_s')}{(1 + \jmath\omega_{base}sT_0')}\right) I_{t(p.u)}e^{\jmath\gamma} \tag{142}$$

From (142),

$$I_{t(p.u)}e^{\jmath\gamma} = \left(\frac{(1 + \jmath\omega_{base}sT_0')}{(R_s - \omega_{base}sX_s'T_0') + \jmath(\omega_{base}sR_sT_0' + X_s)}\right) V_{t(p.u)}e^{\jmath\alpha} \tag{143}$$

The real power input to the motor is

$$P_t = \text{Real}\left(V_{t(p.u)}e^{\jmath\alpha}\left(I_{t(p.u)}e^{\jmath\gamma}\right)^*\right) \tag{144}$$

$$P_t = \left(\frac{(R_s - \omega_{base} s X_s' T_0') + \omega_{base} s T_0' (\omega_{base} s R_s T_0' + X_s)}{(R_s - \omega_{base} s X_s' T_0')^2 + (\omega_{base} s R_s T_0' + X_s)^2} \right) V_{t(p.u)}^2 \quad (145)$$

- If $P_t, Q_t, V_{t(p.u)}$ are defined from Load flow, s can be computed by solving the quadratic equation from (145).
- If s is defined with a terminal voltage, P_t can be computed from (145).
- Similarly, Q_t can be found.