EE549 - Power System Dynamics and Control

Voltage Stability

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Voltage Stability

Voltage stability can be defined as the ability of the system to retain system voltages within acceptable limits when subjected to a disturbance.

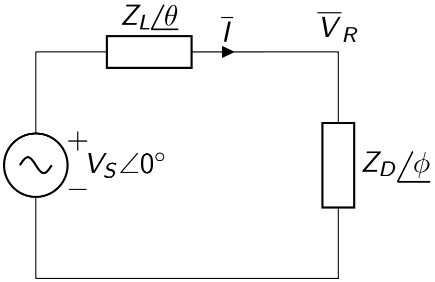
- If the disturbance is large, it is called as large-disturbance voltage stability
- If the disturbance is small, it is called as small-signal voltage stability.
- Voltage stability can be a local phenomenon where only a particular bus or buses in a particular region have voltage stability issue and this may not affect the entire system.
- Voltage stability can be a global phenomenon where many of the system buses experience voltage stability problems which can also trigger angle stability problems and hence can affect the entire system.
- Some of the voltage stability problems can start as a local problem and escalate to global stability problem.

Voltage Stability - Analysis

1. Static Analysis
2. Dynamic Analysis

- In static analysis , the system is assumed to be in steady state and hence instead of taking the DAE of the system only algebraic equations are considered. This type of analysis is suitable for small-disturbances in the system.
- For large disturbances, the DAE are solved and the system response over a certain period of time is observed.
- It is important that for voltage stability the loads should be properly modelled as each type of load will affect the system voltage stability in a different way.
- Similarly the tap changing transformers, shunt or series reactive power compensators, generator reactive power limits, line charging capacitance should be included in the system representation to get an accurate picture of voltage stability.

Basic Concept of Voltage Stability



The magnitude of current is

$$|I| = \left| \frac{V_S}{Z_L\angle\theta + Z_D\angle\phi} \right| = \frac{|V_S|}{\sqrt{Z_L^2 + Z_D^2 + 2Z_L Z_D \cos(\theta - \phi)}} \tag{1}$$

The receiving end voltage magnitude is

$$|V_R| = \frac{|V_S|Z_D}{\sqrt{Z_L^2 + Z_D^2 + 2Z_L Z_D \cos(\theta - \phi)}} \tag{2}$$

The real and reactive power consumed by the load are

$$P_D = \frac{|V_S|^2 Z_D \cos\phi}{\sqrt{Z_L^2 + Z_D^2 + 2Z_L Z_D \cos(\theta - \phi)}} \tag{3}$$

$$Q_D = \frac{|V_S|^2 Z_D \sin\phi}{\sqrt{Z_L^2 + Z_D^2 + 2Z_L Z_D \cos(\theta - \phi)}} \tag{4}$$

- Let $Z_L = 0.25\angle 90^\circ$ and $V_S = 1\angle 0^\circ$.
- The real and reactive power of the load can be increased by decreasing the load impedance Z_D maintaining a constant power factor (0.8).
- The receiving end voltage is plotted with varying real power consumed by the load, with the parameters mentioned here. This curve is called as P-V curve.

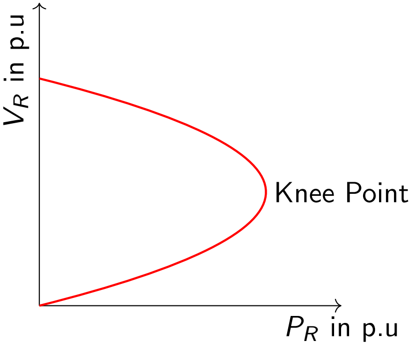
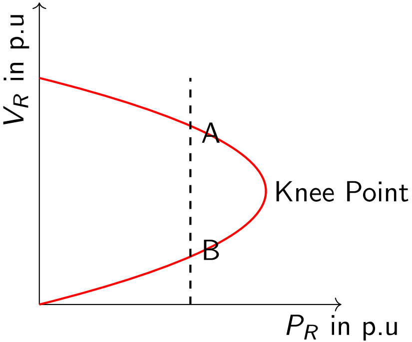


Figure: P-V Curve

Effect of Load Type



- For a constant MVA load, there are two operating points.
- The point A is stable and B is unstable for the constant MVA load.
- In fact for a constant MVA load any operating point below the knee point is unstable.

1. It can be observed from P-V curve that the curve above knee point has $\frac{dV_R}{dP_D}$ negative that is the change in real power and voltage are in the opposite direction, for an increase in power, the voltage decreases and for a decrease in power , the voltage increases.
2. $\frac{dV_R}{dP_D}$ is positive below the knee point.
3. $\frac{dV_R}{dP_D}$ is zero at the knee point is zero.

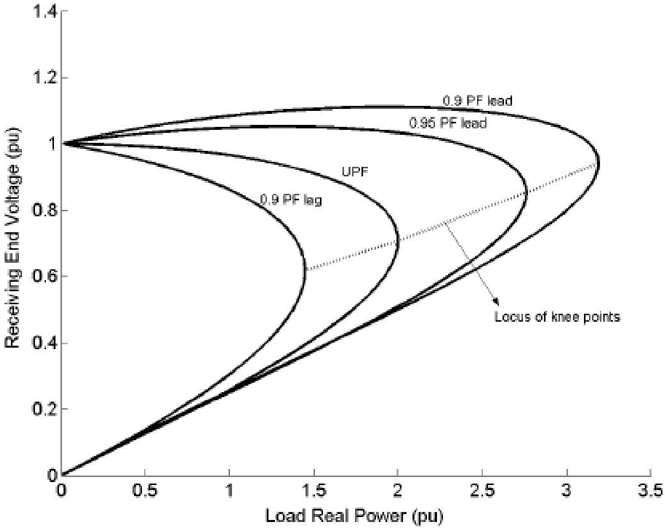


Figure: P-V Curve for different power factor

Source : NPTEL

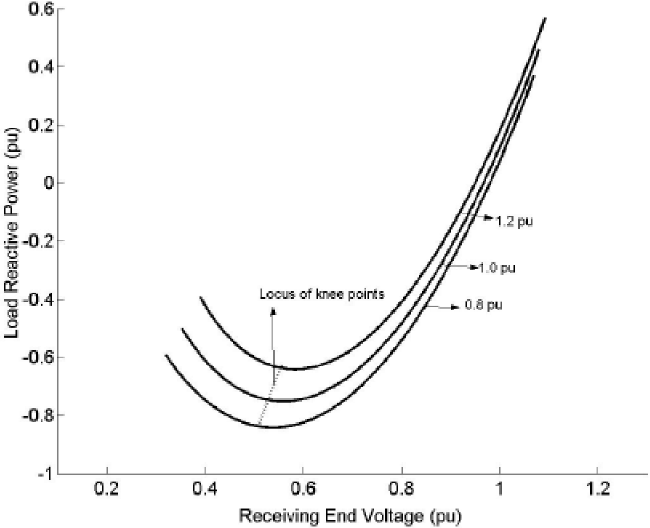


Figure: Q-V Curve for different Real power

Source : NPTEL

- $\frac{dQ}{dV_R}$ is negative to the left of the knee point. This region is unstable.
- $\frac{dQ}{dV_R}$ is positive to the right of the knee point. This region is stable.
- In the unstable region even if reactive power compensation is done through shunt capacitance at the receiving end bus, the voltage of the receiving end bus will not improve.
- One way of finding the voltage stability is to check the sensitivity of each bus voltage with respect to the reactive power injected at that bus and if the sensitivity is positive then it means the operating point is stable and if it is negative then it is unstable.
- This stable region and unstable region are only applicable to constant MVA loads. In case of constant impedance and constant current loads the loads interact with the system and settle at a new operating point as there is no requirement of constant MVA.

Static Analysis

In static analysis, instead of considering all the differential algebraic equations only algebraic power balance equations are considered assuming that the system is in steady state.

There are two methods for assessing whether system is voltage stable or not. They are

1. sensitivity analysis
2. modal analysis

Sensitivity Analysis

The power balance equations are taken assuming that the system is in steady state.

$$P_{Gi} - P_{Di}(V_i) + \sum_{j=1}^{n} V_i V_j Y_{ij} \cos(\alpha_{ij} + \theta_j - \theta_i) = 0 \quad i = 2, \cdots, n \tag{5}$$

$$Q_{Di}(V_i) - \sum_{j=1}^{n} V_i V_j Y_{ij} \sin(\alpha_{ij} + \theta_j - \theta_i) = 0 \quad i = n_g + 1, \cdots, n \tag{6}$$

The linearized form of equations is

$$\begin{bmatrix} \Delta P_2 \\ \vdots \\ \Delta P_n \\ \Delta Q_{n_g+1} \\ \vdots \\ \Delta Q_n \end{bmatrix} = \begin{bmatrix} \mathrm{J}_{P\theta} & \mathrm{J}_{PV} \\ \mathrm{J}_{Q\theta} & \mathrm{J}_{QV} \end{bmatrix} \begin{bmatrix} \Delta \theta_2 \\ \vdots \\ \Delta \theta_n \\ \Delta V_{n_g+1} \\ \vdots \\ \Delta V_n \end{bmatrix}$$

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix} \tag{7}$$

The matrix J is the Jacobian matrix used in Newton- Raphson load flow analysis. For constant real power, ΔP is zero. Therefore,

$$\Delta \theta = -J_{P\theta}^{-1} J_{PV} \Delta V \tag{8}$$

From (8) and (7), we get

$$\Delta Q = [J_{QV} - J_{P\theta}^{-1} J_{PV} J_{Q\theta}] \Delta V \tag{9}$$

Let $J_R = [J_{QV} - J_{P\theta}^{-1} J_{PV} J_{Q\theta}]$.

$$\Delta V = J_R^{-1} \Delta Q \tag{10}$$

- The diagonal elements of J_R^{-1} represent the sensitivity of the voltage with respect to the reactive power injected at that bus.
- Hence, if the V-Q sensitivity of an i^{th} bus is positive then the system is voltage stable and if it is negative then the system is voltage unstable.
- A small positive value of sensitivity indicates that the system is more voltage stable and if the sensitivity is small with negative value then the system is highly voltage unstable.

Modal Analysis

Voltage stability can also be estimated by the eigenvalues and eigenvectors of J_R^{-1} .

J_R can be written as

$$J_R = \Phi \Lambda \Psi \tag{11}$$

where

$$\begin{aligned} \Psi &= \text{ right eigenvector matrix} \\ \Lambda &= \text{ diagonal matrix} \\ \Phi &= \text{ left eigenvector matrix} \end{aligned}$$

Since $\Psi\Phi = I$,

$$J_R^{-1} = \Phi \Lambda^{-1} \Psi \tag{12}$$

Equation (10) can be written as

$$\Delta V = \Phi \Lambda^{-1} \Psi \Delta Q \tag{13}$$

$$\Psi \Delta V = \Lambda^{-1} \Psi \Delta Q \tag{14}$$

Let $v = \Psi \Delta V$ and $q = \Psi \Delta Q$.

$$v = \Lambda^{-1} q \tag{15}$$

$$v_i = \frac{1}{\lambda_i} q_i \tag{16}$$

where v_i is the i^{th} bus modal voltage and q_i is the modal reactive power.

- If the eigenvalue is positive then the system is voltage stable.
- The larger the eigenvalue with positive sign the better the stability.
- If the eigen value has very small positive value then it is very close to $\lambda_i = 0$ that is the critical point or knee point, and hence very close to instability.
- If the eigenvalue is negative then the system is voltage unstable.

Dynamic Analysis

The voltage stability of a system can also be assessed by the transient simulation over a period of time.

$$
\begin{aligned}
\dot{x} &= f(x, I_{dq}, \overline{V}, u) \\
I_{dq} &= h(x, \overline{V}) \\
g(x, I_{dq}, \overline{V}) &= 0
\end{aligned} \tag{17}
$$

- DAE given in (17) should be solved through numerical methods
- The transient simulation should be done for different fault scenarios and the system behaviour should be observed.
- If the system voltages are restored to acceptable values after fault clearing then the system is voltage stable if not the system is voltage unstable.
- The simulations needs to be done for few minutes because the time constant involved can be very small like generator exciter or very large like induction motors or tap changing transformers.
- For voltage stability assessment load modelling is important hence both static and dynamic loads should be modelled.
- The reactive power compensating devices like series / shunt capacitor and static VAr compensator should also be included in the system model.

Small Disturbance Analysis

Small disturbance analysis can also be used for voltage stability analysis. The DAE given in (17) can be represented as

$$\begin{aligned} \dot{\mathrm{x}} &= \mathrm{f}(\mathrm{x}, \mathrm{h}(\mathrm{x}, \overline{\mathrm{V}}), \overline{\mathrm{V}}, \mathrm{u}) \\ \mathrm{g}(\mathrm{x}, \mathrm{h}(\mathrm{x}, \overline{\mathrm{V}}), \overline{\mathrm{V}}, \lambda) &= 0 \end{aligned} \tag{18}$$

where λ is a constant representing the loading of the system, as given below.

$$\begin{aligned} P_D &= P_{D0}(1+\lambda) \\ Q_D &= Q_{D0}(1+\lambda) \end{aligned} \tag{19}$$

P_{D0} and Q_{D0} are the real and reactive power loads at all the load buses for base case.

The linearized form of the system given in (18) is

$$\begin{aligned}\Delta\dot{x} &= \frac{\partial f}{\partial x}\Delta x + \begin{bmatrix}\frac{\partial f}{\partial V} & \frac{\partial f}{\partial \theta}\end{bmatrix}\begin{bmatrix}\Delta V \\ \Delta\theta\end{bmatrix} + \frac{\partial f}{\partial u}\Delta u \\ 0 &= \frac{\partial g}{\partial x}\Delta x + \begin{bmatrix}\frac{\partial g}{\partial V} & \frac{\partial g}{\partial \theta}\end{bmatrix}\begin{bmatrix}\Delta V \\ \Delta\theta\end{bmatrix}\end{aligned} \tag{20}$$

The idea is to increase the loading factor λ in steps and at each step find the eigenvalues of the linearized system

From (20), we can get the following.

$$\begin{aligned}\Delta\dot{x} &= \left[\frac{\partial f}{\partial x} - \begin{bmatrix}\frac{\partial f}{\partial V} & \frac{\partial f}{\partial \theta}\end{bmatrix}\begin{bmatrix}\frac{\partial g}{\partial V} & \frac{\partial g}{\partial \theta}\end{bmatrix}^{-1}\frac{\partial g}{\partial x}\right]\Delta x + \frac{\partial f}{\partial u}\Delta u \\ &= J_{Sys}\Delta x + \frac{\partial f}{\partial u}\Delta u\end{aligned} \tag{21}$$

- J_{Sys} is called as the system Jacobian and is different from the load flow Jacobian.
- The eigenvalue of J_{Sys} for different loading conditions, obtained by varying the loading factor λ, are computed.
- If one of the eigenvalues becomes zero or a pair of complex conjugate eigenvalues is on the imaginary axis for a particular loading factor λ then the load corresponding to the loading factor λ is a critical load at which the system becomes voltage unstable.
- If a complex conjugate pair of eigenvalues is on imaginary axis then that operating point is called as Hopf-bifurcation point.
- If one of the eigenvalues is on the origin then that operating point is called as saddle node point. The saddle node bifurcation point represents the knee point.
- Hopf-bifurcation point occurs at a load which is less than the load at which the saddle node bifurcation point occurs. Hence, due to the dynamics involved the system can become voltage unstable even before the operating point reaches the knee point.